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DISTRIBUTION OF THE MEANS OF SAMPLES OF n DRAWN AT RANDOM FROM A POPULATION REPRESENTED BY A GRAM-CHARLIER SERIES

By

G. A. Baker

The use of a Gram-Charlier series for the representation of the frequencies of uni-variate populations has been recommended for a long time by Gram, Charlier, Thiele, Edgeworth, Bowley and Arne Fisher. In practice it has been found that, in many cases, the first two or three terms give a fairly adequate representation of many populations. The arithmetic mean is one of the most used statistical constants. Thus it is of considerable practical and theoretical interest to be able to specify the distribution of the means of samples of n drawn from a population represented by a Gram-Charlier series. It is the object of this paper to obtain the distributions of the means of such samples exactly.

J. O. Irwin[1] has given a formal development for obtaining the distribution of the totals, i. e. n times the mean, of samples of n drawn at random from any continuous population as the solution of an integral equation, if a solution exists.

That is, if the population is represented by $f(x)$, $a \leq x \leq b$, $f(x)$ being continuous, then the distribution $\psi(x)$ of the totals of samples of n drawn at random from $f(x)$ is given by the solution of

$$(1) \qquad F(\alpha) = \int_{na}^{nb} \psi(x)\, e^{\alpha x}\, dx$$

where

$$(2) \qquad F(\alpha) = \left[\int_a^b f(x)\, e^{\alpha x}\, dx\right]^n$$

Interpretate α as a complex variable and assume that (1) is valid along the ray $\alpha = i\beta$, where β is real, that $X(x) = \psi(x)$, $na \leq x \leq nb$ and zero outside of these limits, then an applica-

tion of Fourier's Integral Theorem gives

$$(3) \qquad X(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(i\beta)\, e^{-i\beta x}\, d\beta$$

or

$$(4) \qquad \psi(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(i\beta)\, e^{-i\beta x}\, d\beta, \qquad na \leq x \leq nb\,,$$

provided that $X(x)$ can be shown, independently, to vanish $na > x > nb$. The distribution of the means of samples of n can be obtained from $\psi(x)$ by means of the transformation

$$(5) \qquad X = n\bar{x}$$

After considerable formal computation, the distribution of the means of n from a population represented by a Gram-Charlier series can be obtained as a solution of (1), and the result may be stated in the following theorem.

THEOREM I. If a population can be represented by the first $m-1$ terms of a Gram-Charlier series, i. e.

$$(6) \qquad f(x) = a_0\, \phi_0(x) \quad a_3\, \phi_3(x) + \cdots + a_m\, \phi_m(x)$$

where

$$(7) \qquad \phi_i(x) = \frac{d^i\left(e^{-\frac{1}{2}x^2}\right)}{dx^i}$$

then the means of samples of n drawn at random from the population represented by (6) will be distributed as proportional to

$$(8) \quad \sum \frac{n!}{V_b!\, V_3! \cdots V_m!}\, a_0^{V_b}\, a_3^{V_3} \cdots a_m^{V_m}\, \frac{d^{\,m\sqrt{m} + (m-1)\sqrt{m-1} + \cdots + 3\sqrt{3}}\left(e^{-\frac{nx^2}{2}}\right)}{d(nx)^{\,m\sqrt{m} + m-1\sqrt{m-1} + \cdots + 3\sqrt{3}}}$$

summed for all non-negative integral solutions of

$$(9) \qquad V_b + V_3 + V_4 + \cdots + V_m = n$$

The plan of the proof will be to prove (8) for the case $m = 3$ and then complete the proof by mathematical induction.

Suppose a population may be represented by

$$(10)\quad f(x) = a_0 e^{-\frac{1}{2}x^2} + a_3\left[-x^3+3x\right]e^{-\frac{1}{2}x^2}$$

$$-\infty \leq x \leq \infty$$

Then $F(\alpha)$ defined by (2) is

$$(11)\quad F(\alpha) = \left[\int_{-\infty}^{\infty}\left[a_0 + a_3(-x^3+3x)\right]e^{-\frac{1}{2}x^2} e^{\alpha x}\,dx\right]^n$$

Put

$$x - \alpha = y$$

Then

$$F(\alpha) = \left[e^{\frac{\alpha^2}{2}}\int_{-\infty}^{\infty}\left[a_0 + a_3(-y^3 - 3\alpha y^2 - 3\alpha^2 y\right.\right.$$

$$(12)\quad \left.\left. -\alpha^3 + 3y + 3\alpha\right]e^{-\frac{1}{2}y^2}\,dy\right]^n$$

$$= \left(\sqrt{2\pi}\right)^n e^{\frac{n\alpha^2}{2}}\left[a_0 - a_3\alpha^3\right]^n$$

Thus

$$(13)\quad F(i\beta) = (\sqrt{2\pi})^n e^{-\frac{n\beta^2}{2}}\left[a_0 + a_3 i\beta^3\right]^n$$

The assumptions leading to (4) are satisfied, whence

$$(14)\quad \psi(x) = \frac{(\sqrt{2\pi})^n}{2\pi}\int_{-\infty}^{\infty} e^{-\frac{n\beta^2}{2}}\left[a_0 + a_3 i\beta^3\right]^n e^{-i\beta x}\,d\beta$$

The $s+1^{th}$ term of (14) is

$$(15)\quad \frac{(\sqrt{2\pi})^n}{2\pi}\binom{n}{s}\int_{-\infty}^{\infty} a_0^{n-s} a_3^s (i)^s \beta^{3s} e^{-\frac{n\beta^2}{2}} e^{-i\beta x}\,d\beta$$

Now

(16) $$e^{-i\beta x} = \cos\beta x - i\sin\beta x$$

If s is odd, i. e. $s = 2a+1$,

$$\int_{-\infty}^{\infty} \beta^{3s} \cos\beta x \, e^{-\frac{1}{2}n\beta^2} d\beta$$

vanishes, because it is an odd function, and there is left

(17) $$\frac{(\sqrt{2\pi})^n}{2\pi}\binom{n}{s}\int_{-\infty}^{\infty} a_0^{n-s} a_3^s (i)^{2a+1} (-1) i\beta^{3(2a+1)} e^{-\frac{n\beta^2}{2}} \sin\beta x \, d\beta$$

It is known that

$$\int_{-\infty}^{\infty} e^{-a^2x^2}(\sin bx) x^{2c+1} dx = (-1)^{c+1}\frac{\sqrt{\pi}}{a}\frac{d^{2c+1}\left(e^{-\frac{b^2}{4a^2}}\right)}{db^{2c+1}}$$

Hence (17) becomes

(18) $$\frac{(\sqrt{2\pi})^{n-1}}{2\pi\sqrt{n}}\binom{n}{s} a_0^{n-s} a_3^s \frac{d^{3s}\left(e^{-\frac{x^2}{2n}}\right)}{dx^{3s}}$$

If s is even, i. e. $s = 2a$ (15) becomes

(19) $$\frac{(\sqrt{2\pi})^n}{2\pi}\binom{n}{s}\int_{-\infty}^{\infty} a_0^{n-s} a_3^s (i)^{2a} \beta^{2a} e^{-\frac{1}{2}n\beta^2} \cos\beta x \, d\beta$$

because the sine is an odd function.

It is known that

(20) $$\int_{-\infty}^{\infty} e^{-a^2x^2}(\cos bx) x^{2c} dx = (-1)^c \frac{\pi}{a}\frac{d^{2c}\left(e^{-\frac{b^2}{4a^2}}\right)}{db^{2c}}$$

whence (19) becomes

(21) $$\frac{(\sqrt{2\pi})^{n+1}}{2\pi\sqrt{n}}\binom{n}{s} a_0^{n-s} a_3^s \frac{d^{3s}\left(e^{-\frac{x^2}{2n}}\right)}{dx^{3s}}$$

as before.

Thus the totals of samples of n drawn at random from a population represented by (10) are distributed as proportional to

$$(22) \qquad \sum_{s=0}^{s=n} \binom{n}{s} a_0^{\,n-s} a_3^{\,s} \, \frac{d^{3s}\left(e^{-\frac{x^2}{2n}}\right)}{dx^{3s}}$$

or the distribution of the means is proportional to

$$(23) \qquad \sum_{s=0}^{s=n} \binom{n}{s} a_0^{\,n-s} a_3^{\,s} \, \frac{d^{3s}\left(e^{-\frac{nx^2}{2}}\right)}{d(nx)^{3s}}$$

as given by the theorem.

It is apparent from the proof for the case $m=3$ that to complete the proof for the general case it is only necessary to show that

$$(24) \qquad \int_{-\infty}^{\infty} H_m(y+\alpha) \frac{e^{-\frac{1}{2}y^2}}{\sqrt{2\pi}} \, dy = \alpha^m$$

if $m>3$ it being noted that the negative sign that arises in the differentiation of $\phi_0(x)$ and that is omitted in $H_m(x)$ (the Hermite polynomial of degree m) is automatically taken care of.

Relation (24) has been proven true for the case $m=3$. By actual computation, it may be shown to be true $m=1$ and $m=2$ also. Assume (24) to be true for $m=k-1$, then if it can be shown that (24) is true for $m=k$ that will complete the proof.

Thus, it is assumed that

$$(25) \qquad \int_{-\infty}^{\infty} H_{k-1}(y+\alpha) \frac{e^{-\frac{1}{2}y^2}}{\sqrt{2\pi}} \, dy = \alpha^{k-1}$$

For the moment, assume that (24) is not true for $m=k$ and differentiate it with respect to α, the conditions being sufficient for differentiating under the integral sign. Thus (24) becomes

$$(26) \qquad \int_{-\infty}^{\infty} H_k'(y+\alpha) \frac{e^{-\frac{1}{2}y^2}}{\sqrt{2\pi}} \, dy \neq k\alpha^{k-1}$$

Multiply (25) by k and subtract it from (26), thus

$$(27) \qquad \int_{-\infty}^{\infty} \left[H_k'(y+\alpha) - k H_{k-1}(y+\alpha)\right] \frac{e^{-\frac{1}{2}y^2}}{\sqrt{2\pi}}\, dy \neq 0$$

But

$$H_k'(y+\alpha) - k H_{k-1}(y+\alpha) = 0$$

Thus there is a contradiction from which it follows that (24) is true for $m = k$ if (25) is true.

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G. A. Baker

THE USE OF LINEAR FUNCTIONS TO DETECT HIDDEN PERIODS IN DATA SEPARATED INTO SMALL SETS

By

EDWARD L. DODD

I.—INTRODUCTION

Readers who have access to the Handbook of Mathematical Statistics[1] will find in chapter XI a synopsis of a periodogram analysis by W. L. Crum, with references to some of the important papers on period testing.

My own interest in this subject was aroused several years ago by Dr. J. A. Udden,[2] Director of the Bureau of Economic Geology at the University of Texas, who had in his possession measurements of the thicknesses of successive layers of anhydrite ($CaSO_4$) taken from a Texas oil well. The material, Dr. Udden noted, was "suggestive of cycles" (p. 351); but one difficulty was mentioned: "Probably 2 per cent of the layers are indistinct." It was not always possible to tell whether the number recorded as the thickness of a layer represents a single deposit or two or more deposits insufficiently separated by the usual bituminous demarcation. The analogous difficulty in distinguishing consecutive rings of big trees[3] was met by comparison of the rings of trees from the same forest. But such companion records were not available for the rock lamina.

A little reflection will show that the usual method of testing

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1 Rietz, Houghton Mifflin Co., 1924.

2 "Laminated Anhydrite in Texas." *Bulletin of the Geological Society of America*, Vol. 35 (1924), pp. 347-354.

3 A. E. Douglass, "Climatic Cycles and Tree Growth," Carnegie Institution of Washington, Publication No. 289 (1919).

for cycles, from data arranged in columns becomes vitiated if in several instances merging of layers has taken place—not so much because of the exaggerated size of certain items, but because the items get into the wrong columns. When a step is lost, all subsequent items are misplaced.

My purpose is then to explain how tests for periods can be made by first using the data in small sets—thus minimizing the vicious effects of a merger—and then by suitably combining the results obtained from these small sets.

We might as well admit at the start that a demonstration of a periodicity is in general impossible. Perhaps the revolution of the earth on its axis represents a demonstrated periodicity. But for the most part, announced periodicities are merely improbable or probable. There is no absolute proof that they exist. We know that what we call "pure chance," typified by the throws of a coin, will sometimes yield irregularities of oscillation between two states, the minimum and the maximum, which to all appearances is a "periodicity." The question arises: About how often will pure chance thus deceive us? All we can do is to compute certain relative frequencies or probabilities. If the probability found is very, very small, that the apparent periodicity had its origin in pure chance, we assert with some assurance that a real periodicity exists. In this mode of approach, this paper will follow rather closely Arthur Schuster,[1] whose work is fundamental.

Although Schuster's main interest was in the quadratic function, "intensity"—at first, in the square root of intensity, Terrestrial Magnetism, loc. cit.—he pointed out (p. 27) how certain con-

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1 "On the investigation of hidden periodicities with application to a supposed 26-day period of meteorological phenomena." *Terrestrial Magnetism,* Vol. 3 (1898), pp. 13-41. In my paper, "The probability law for the intensity of a trial period, with data subject to the Gaussian law," *Bulletin of the American Mathematical Society,* Vol. 33 (1927), pp. 681-684, I referred to Schuster's paper in the *Proceedings of the Royal Society of London.* Reference should have been made also to the above paper in *Terrestrial Magnetism,* where the probability law is given for the square root of intensity (p. 21), which can easily be thrown into the form given in my paper. Schuster, however, postulated (p. 20) that "2π p is a submultiple of a right angle"—a condition which would not always be satisfied—also (p. 21) that the vectors be distributed according to the law of errors centered at the *origin,* an inconvenient restriction, and his method did not bring out the different law of distribution needed for the case when the period is equal to two.

clusions could be reached through integrals—substantially linear functions, if integration is regarded as summation. It is this approach to period testing through linear functions that I am setting forth in his paper. Some special attention must be given to phase in the application of this method.

Most of the methods for detecting periodicities make use of the trigonometric functions, with their well known properties, in particular, use is made of the Sines and cosines of an angle and its multiples, as in harmonic analysis and Fourier series. With the aid of these harmonic multipliers, linear functions are first formed; and from these, by squaring and adding, a quadratic function, which plays the central role, as "intensity." In the method set forth in this paper, however, the linear functions themselves are the most important, not merely for graphical representation, but for determining probabilities.

Suppose, then, that a set of numbers is furnished us—perhaps from an unknown source—for example a set of ten numbers consisting of 5's and 1's alternating:

$$5,\ 1,\ 5,\ 1,\ 5,\ 1,\ 5,\ 1,\ 5,\ 1.$$

Has this set of numbers the period *two*? If this question means: Is there a function of period *two* which takes on these ten values, the answer is: Yes, namely—

$$3 + 2 \cos \pi r \qquad\qquad r = 0, 1, 2, \ldots . 9$$

Here, as usual, π means $180°$, obtained from a complete revolution of $360°$ by dividing by *two*. If, in place of an integer r, we take a continuous variable x, and plot

$$y = 3 + 2 \cos \pi x$$

from $x = 0$ to $x = 10$, a wave curve is formed with each upper crest at 5, and each depression at 1.

But usually in period testing, something is desired beyond the mere possibility of making a mathematical curve fit the data. Perhaps a farmer on each 10 acres of his farm has raised 5 bales of cotton, 1 bale of cotton, 5 bales of cotton, etc., alternately for 10 years, under apparently the same conditions as to labor, fer-

tilizer, etc. He would like to know whether this is due to mere chance or to some recurrence at two-year intervals of droughts, pests, or adverse conditions. Stranger events do, indeed, occur by pure chance than the foregoing hypothetical yield of cotton. But the regularity postulated above would strike almost anyone as exceptional, and it would be prudent for our farmer to believe that there was some non-fortuitous cause of the regularity, and to try to discover it.

Let us, indeed, set up a chance situation to correspond to the foregoing yield of cotton. If the two faces of a coin are marked 5 and 1, and are recorded as such, the probability for ten throws starting with 5 and alternating between 1 and 5 is only 1/1024. A bet of $1,023 against $1 would measure the unusualness of the specified succession of 5's and 1's.

That this occurrence is unusual may be signalized by another test and method of approach. Let X_r denote the result of the r th trial of an independent chance variable, which with equal likelihood ($p_1 = 1/2 = p_2$) takes on the values 5 or 1, and can take on no other value. The "mean value" of X_r is then, by definition,

$$p_1(5) + p_2(1) = \tfrac{1}{2}(5) + \tfrac{1}{2}(1) = 3$$

This would, indeed, be also the average value of the five 5's and five 1's in the illustration. The "mean error" ε of X_r would be found from

$$\varepsilon^2 = \frac{1}{2}(5-3)^2 + \frac{1}{2}(1-3)^2 = 4 \,, \qquad \varepsilon = 2$$

This would be also the standard deviation σ of the numbers in the illustration—that is

$$\sigma^2 = \frac{1}{10}\left[(5-3)^2 + (1-3)^2 + (5-3)^2 + \cdots + (1-3)^2\right] = 4 \,, \qquad \sigma = 2$$

Now let

$$X = X_1 - X_2 + X_3 - \cdots - X_{10}$$

Since the signs alternate, the mean value of X is zero; since

there are ten terms, the mean error of X is $\epsilon\sqrt{10} = 2(3.16) = 6.32$. If now X_r should take on alternately the values of 5 and 1, then X would become 20. It would thus exceed its mean value zero by more than three times its mean error, or more than four and one-half times its "probable error." This is commonly regarded as "significant."

To see a little more clearly into the mechanism of the above result, let us pass from the numbers X_r to their deviations from their mean value 3.

Let

$$x_1 = X_1 - 3\,, \quad x_2 = X_2 - 3\,, \quad \cdots\,, \quad x_r = X_r - 3\,, \quad \cdots$$

Then the mean value of x_r is zero, and its mean error is 2. Now let

$$x = x_1 - x_2 + x_3 - \quad \cdots - x_{10}$$

Then the mean value of x is zero and its mean error is $\epsilon\sqrt{10}$; in both respects it resembles X. Furthermore it takes on the same value 20 that X takes on when the 5 and 1 alternate; since $x_1 - x_2 = (X_1 - 3) - (X_2 - 3) = X_1 - X_2$, etc. And here again 20 is a remarkable value for x since it represents an excess of more than three times its mean error. But let us now find x directly from the values taken on by x_r, when X_r alternates between 5 and 1.

$$x = 1\ (2) - 1\ (-2) + 1\ (2) - \ldots -1\ (-2) = 20$$

The feature to be noted is that the successive values of x_r and of $\cos \pi (r-1)$ match in sign, for $r = 1, 2, 3 \ldots 10$.

$$x_r = 2,\ -2,\ 2,\ -2,\ \cdots\ ,\ -2$$

$$\cos \pi(r-1) = 1,\ -1,\ 1,\ -1,\ \cdots\ ,\ -1$$

Each product $x_r \cos \pi (r-1)$ is then positive; and this accounts for the large value of x. This matching in sign of the deviations of the data with the successive terms of a test function $\cos 2\pi(r-1)/k$ or perhaps $\cos 2\pi r/k$ when k is given

a particular value—here $k = 2$—is, indeed, fundamental. Also the similarity between the properties of X and x will be found to be maintained in more general cases.

The foregoing illustrates the method of period testing to be set forth in this paper. A general assumption is at first made, that the data contain no periodic constituent, but on the contrary represent mere chance fluctuations. Certain linear functions of the data are found with coefficients which are the cosines or sines of multiples of the angle associated with a given period. For these functions, the fluctuations usually to be expected are to be computed—assuming that the measurements represent chance data. If the actual values which these functions take on are greatly in excess of what is expected of them, the initial assumption that the data are due to chance is called into question. It may be more reasonable to suppose that to some extent the data conform to the period associated with the cosine multipliers involved in the test. These "harmonic" multipliers, indeed, pass through a succession of positive and negative values in a regular way. If the positive and negative fluctuations of the measurements from their average value are well "timed" with those of the harmonic multipliers, we get a sum of products nearly all positive, thus a much larger result than if positive numbers were not matched with positive, negative with negative numbers.

As preliminary to all tests, the data may be divided into fairly large groups of consecutive measurements—say with 120 measurements in a group; for 120 is a multiple of 2, 3, 4, 5, 6, 8, 10, 12, 15, 20, 24, 40, 60, numbers quite suitable for trial periods. The arithmetic mean and standard deviation of each such group may be computed. These may usually be accepted as close approximations to the mean value and mean error of the measurements of the group.

To illustrate further the nature of the tests to be applied, let us imagine that the 120 measurements of a group are recorded on slips of papers, these slips put into a bag, drawn at random, and recorded as drawn. This set of numbers would have the same arithmetic mean and standard deviation, noted above, no matter in what order they are drawn and recorded. But periodicities depend upon the order of the measurements. A chance order of measurements x_r, such as established by drawing from a bag, would very seldom match sufficiently well a periodic function like

cos $2\pi r/2$ with period 2, or cos $2\pi r/3$ with period 3, etc., to make a test function $C(k) = \Sigma X_r$ cos $2\pi r/k$ noticeably large. Thus, if for some particuler k, the function $C(k)$, computed from the data in their actual given order, turns out to be significantly large, the indication is that the data contain a constituent with period k.

We mean here that each measurement of the set may be thought of as the sum of certain constituents, one of which is periodic with period k. Another constituent may perhaps have a different period k'. Still another constituent may be a chance variable with no regularity which can properly be called periodic.

II. Trigonometric Formulas.

Of considerable use are the simple formulas:

(1) $$\sin a \sin b = \frac{1}{2}\cos(a-b) - \frac{1}{2}\cos(a+b)$$

(2) $$\cos a \cos b = -\frac{1}{2}\cos(a-b) + \frac{1}{2}\cos(a+b)$$

Indeed, by using (1) in summing the product sin $(r\theta + \alpha)$ sin $\theta/2$ from $r = 0$ to $(n-1)$ there is obtained,[1] in case θ is not a multiple of 360°,

(3) $$\sum_{r=0}^{n-1} \sin(r\theta + \alpha) = \sin\left(\frac{(n-1)\theta}{2}\ \alpha\right) \frac{\sin n\theta/2}{\sin \theta/2}$$

Likewise, for $\theta \not\equiv 0 \pmod{360°}$; i e., θ not a multiple of 360°,

(4) $$\sum_{r=0}^{n-1} \cos(r\theta + \alpha) = \cos\left(\frac{(n-1)\theta}{2} + \alpha\right) \frac{\sin n\theta/2}{\sin \theta/2}$$

As important special cases, we have when $n\theta$ is a multiple of 360°,

1 For formulas suitable for period testing and for a historical review of this subject with references, the reader is referred to the article of H. Burkhardt in *Encyklopädie der Mathematischen Wissenschaften*, II A 9a, pp. 642-694.

$$(5)\qquad \sum_{r=0}^{n-1} \sin(r\theta+\alpha) = 0 = \sum_{r=0}^{n-1} \cos(r\theta+\alpha), \quad \begin{matrix} \theta \not\equiv 0 \ (\text{mod } 360°) \\ n\theta \equiv 0 \ (\text{mod } 360°) \end{matrix}$$

As an application of (5), let $X_1, X_2, \cdots X_r \cdots X_n$ be any set of n numbers; let C be any constant. Then

$$(6)\qquad \sum_{r=0}^{n-1} (X_r - C)\cos(r\theta+\alpha) = \sum_{r=0}^{n-1} X_r \cos(r\theta+\alpha), \quad \begin{matrix} \theta \not\equiv 0 \ (\text{mod } 360°) \\ n\theta \equiv 0 \ (\text{mod } 360°) \end{matrix}$$

Likewise for sin ($r\theta+\alpha$).

The above signifies that if the X_r represent data to be subjected to tests with harmonic multipliers, where an integral number of complete cycles is taken, it is immaterial where the origin for the data is taken. In the theory, the C will be usually taken as the arithmetic mean of the data; in computation, the C may be some simple number which will reduce the number of significant figures in the data.

III. Chance Data in Distributions with close contact at extremities

Chance data distributed normally will be considered first. Given n numbers or variates $X_1, X_2, \ldots, X_n$, the arithmetic mean M and standard deviation σ are determined by

$$(7)\qquad M = \frac{1}{n}(X_1 + X_2 + \cdots + X_n)\ ;\ \sigma^2 = \frac{1}{n}\left[(X_1 - M)^2 + \cdots (X_n - M)^2\right].$$

Let

$$(8)\qquad \phi(t) = \frac{2}{\sqrt{2\pi}}\int_0^t e^{-\frac{t^2}{2}}\,dt$$

The data will be said to be normally distributed if the number of variates lying between $M+\lambda_1$ and $M+\lambda_2$ is approximately

$\frac{n}{2}\left[\phi(\lambda_2/\sigma)-\phi(\lambda_1/\sigma)\right]$ for all values of $\lambda_1 < \lambda_2$. Here n is supposed to be at least moderately large. To express this in the language of probability, suppose the n numbers X_1, X_2, are recorded on slips and put into a bag, and suppose a slip is drawn out. Then, for the X_r thus drawn—

(9) Probability that $\lambda < X_r < \lambda + d\lambda$ is $\frac{d\lambda}{\sigma\sqrt{2\pi}} e^{-(\lambda-M)^2/2\sigma^2}$

where, if $d\lambda$ is taken rather small, the n is to be thought of as rather large.

The important theorem needed here—substantially explained, if not proven, in most books on probability—is that if sets of k of these variates are drawn at random, and linear functions with fixed coefficients, such as

(10) $$F(k) = a_1 X_1 + a_2 X_2 + \cdots + a_k X_k$$

are formed, these functions $F(k)$ as determined in sets of drawings will be normally distributed with standard deviation σ_k, where

(11) $$\sigma_k^2 = \sigma^2(a_1^2 + a_2^2 + \cdots + a_k^2) .$$

If, in particular $a_r = \cos(r\theta + \alpha)$ making $2a_r^2 = 1 + \cos(2r\theta + 2\alpha)$ and if further $k\theta = 360°$ with $k > 2$, it follows from (5) that

(12) $$\sigma_k^2 = k\sigma^2/2$$

Let us now in (6) set $C = M$; or rather, what amounts to the same thing, change the origin for the data so as to make $M = 0$. Then the "mean value" or "expected value" of $F(k)$ in (10) is zero. Then, with the use of (8) and (12), it follows that

(13) Probability that $|F(k)| > 3\sigma\sqrt{k/2}$ is $1 - \phi(3)$ $= 0.0027$.

This small probability by no means implies impossibility. However, if the computed $|F(k)|$ exceeds $3\sigma\sqrt{k/2}$, there

is some ground for doubting the original hypothesis that the data under consideration exhibit a chance arrangement. Sometimes such evidence gathered from different sections of the data can be made cumulative. A comparatively large value for $F(k)$ in (10) is likely to result when the signs of the X_r match the signs of the a_r, taken as in (11) and (12) as cos ($r\theta + \alpha$), giving a cycle or period of k items.

To what extent evidence is thus afforded for the specific period of k needs further consideration. But, until we have found an adequate number of instances in which some inequality like (13) is satisfied we have obtained little evidence of any periodicity at all.

Thus far we have considered normally distributed data, conforming to the well-known symmetric bell-shaped probability curve. But this is more restrictive than necessary. Results substantially the same can usually be obtained for distributions—even those not symmetric and not mesokurtic—which at both ends taper off in slender tails. Although the particular numerical value of the probability given in (13) is no longer applicable to these curves, the probability nevertheless is usually very small, as presented geometrically as slices of the two tails.

Moreover, the equations (11) and (12) arise from the general theory of expected values. Suppose that p_r is the probability that the chance variable X will take on the value ξ_i, where $p_1 + p_2 + \cdots + p_s = 1$. Then the expected value of X is, by definition

$$(14) \qquad E(X) = p_1 \xi_1 + p_2 \xi_2 + \cdots + p_s \xi_s = E_1 ,$$

and its mean error $\varepsilon(X)$ is defined by

$$(15) \qquad \varepsilon^2(X) = p_1 (\xi_1 - E_1)^2 + \cdots + p_s (\xi_s - E_1)^2 = E(X - E_1)^2$$

It is common to identify expected value and mean error with arithmetic mean and standard deviation as approximations. In applying (6), the supposition was made that the origin be taken so as to make $M = 0$. With this adjustment, we may take $E(X) = 0 = E$, in (14) and (15). As the X_j are regarded as independent, the theory of expected values applied to (10) leads first from $E(X) = 0$ to $E\left[F(k)\right] = 0$, as mentioned before;

and then to (11)—noting that when $i \neq j$, $E(X_i X_j) = 0$, in the expression for $E\left[F(k) - O\right]^2$.

IV. Data with Periodic Constituents.

We now consider data of the form

$$(16) \qquad W_r = X_r + Y_r + Z_r \,,$$

where X_r is, as before, a chance variable; but

$$(17) \qquad Y_r = b\cos(r\theta + \beta) \; ; \quad Z_r = c\cos(r\theta' + \gamma) \,,$$

$$(18) \qquad k\theta = 2\pi = 360^\circ = k'\theta'$$

Here Y_r and Z_r are periodic with periods k and k', not necessarily integral, amplitudes b an c, phases β and γ, respectively. Dealing first with Y_r, let m and n be whole numbers such that $n = mk$. Then, in analogy with (10), but applied to n of the Y's take

$$(19) \qquad F(n) = \sum_{r=0}^{n-1} Y_r \cos(r\theta + \alpha) = \frac{nb}{2}\cos(\alpha - \beta) \; ; \quad k > 2$$

as may be shown from (2) and (5). The magnitude of $F(n)$ depends materially upon the phase difference $(\alpha - \beta)$. But

$$(20) \qquad \left|\cos(\alpha - \beta)\right| > 0.92 \,, \quad \text{if } \left|\alpha - \beta\right| \leqq 22\tfrac{1}{2}^\circ$$

Thus if the phase α of the test function cos ($r\theta + \alpha$) differs from the phase β of the data, taken now as Y_r in (17), by not more than 22½°, the absolue value of $F(n)$ in (19) will fall below its maximum, $nb/2$, by less than 8%. The phase β of the Y constituent of data would in general be unknown; but if for α we take *eight* consecutive multiples of 45°, one of these would fall within 22½° of any designated angle β, (mod 360°). Moreover, if in (19), α is increased by 180°, $F(n)$ merely changes its sign, and thus gives no essentially new information. Hence, instead of *eight* multiples of

45°, the four multiples—90°, 0°, –45°, 45°—will be adequate. These, taken in the above order, give

$$(21)\qquad S=\sum_{r=0}^{n-1} Y_r \sin r\theta \qquad ; \quad C=\sum_{r=0}^{n-1} Y_r \cos r\theta$$

$$(22)\qquad S'=\sum_{r=0}^{n-1} Y_r \sin(r\theta+45^\circ)\ ; \quad C'=\sum_{r=0}^{n-1} Y_r \cos(r\theta+45^\circ) .$$

Furthermore, it is not necessary to compute S' and C' in (22) directly from the data, since

$$(23)\qquad S'=\frac{\sqrt{2}}{2}(C+S) \quad ; \qquad C'=\frac{\sqrt{2}}{2}(C-S) \quad ;$$

but a direct computation of S' or C' would serve well as a check upon (21). Thus, if in (19) we assign to α the four values mentioned above, we get S, C, S', C', in (21), (22) such that for one of these quantities (20) is satisfied, which makes $F(n)$ in (19) take a value almost equal to $nb/2$. This increases as n itself—not merely as the square root of n, an increase typical for $\sum X_r \cos(r\theta+\alpha)$, see (12), (16), with k replaced by n.

Let us now consider the function:

$$(24)\qquad G(n)=\sum_{r=0}^{n-1} Z_r \cos(r\theta+\alpha)=c\sum_{r=0}^{n-1}\cos(r\theta+\alpha)\cdot\cos(r\theta'+\gamma).$$

By (2), the terms above have the form

$$(25)\qquad \frac{c}{2}\cos\left[r(\theta+\theta')+\alpha+\gamma\right]+\frac{c}{2}\cos\left[r(\theta-\theta')+\alpha-\gamma\right] .$$

In order to use (5), we postulate that neither $\theta+\theta'$ nor $\theta-\theta'$ is zero or any other multiple of 360°, in particular $\theta'\neq\theta$. With $n=mk$, as before, (18) gives $n\theta=mk\theta=m(2\pi)$. Hence, it follows that

$$\sin n(\theta+\theta')/2=\pm\sin n\theta'/2=\pm\sin mk\pi/k' .$$

Likewise, $\sin n\,(\theta-\theta')/2 = \pm \sin mk\pi/k'$.

Hence, from (4), (25) it follows that $G(n)$ in (24) contains the factor sin $mk\pi/k$. Thus $G(n) = 0$, if

(26) $k' = mk,\ \frac{mk}{2},\ \frac{mk}{3},\ \ldots\ \frac{mk}{m-1},\ \frac{mk}{m+1},\ \frac{mk}{m+2}, \ldots$

This may also be written

(27) $qk' = mk$, $q =$ any whole number $\neq m$.

Thus, if m cycles of a period k are used as multipliers in the form (24) upon a set of mk numbers Z_r with period $k' = mk/q$, where q is any whole number except m, the result is zero. It should be noted that in order to apply (4) to (24) (25), *to get* (26), it was necessary to require that $k' \neq k$, which would make $q \neq m$ in (27). To illustrate: 3 cycles, each with period $k = 4$, will "annihilate" a set of 12 numbers if these are the successive terms of $C\cos(\gamma + 2\pi r/k')$ with period k' equal to 12, or 12/2, or 12/4, or 12/5, etc., but not 12/3.

Indeed, $G(n)$, instead of vanishing when k' is set equal to k in (24), making $\theta' = \theta$, takes on just about its maximum value $nc/2$ in this case when the phase α is properly chosen—see (19), (20). Inasmuch as $G(n)$ in (24) is a continuous function of θ', it follows that if k' is taken very close to k, $G(n)$ would be almost as large as for $k' = k$. But from (26) we learn that $G(n)$ goes down to zero if k' is allowed to be as small as $mk/(m+1)$ or as large as $mk\ (m-1)$.

Thus, if significantly large results are obtained when using the test function cos $(r\theta + \alpha)$ with period $k = 2\pi/\theta$, the individual period k itself is not necessarily indicated. But rather, the test furnishes evidence that *some period close to k is present in the data*, this proximity being expressed by the inequality (see 26)

(28) $\frac{m}{m+1}k < k' < \frac{m}{m-1}k$.

The relations involved here can perhaps be set forth in greatest simplicity by using integration to effect summations—cf. (34). In the test function cos $(r\theta + \alpha)$, set the phase $\alpha = 0$, and take $x = r\theta$, where $\theta = 2\pi/k$. Suppose k is rational, and take an even integer m such that $n = mk$ is an even integer. Consider the test as covering the data, from $x = -m\pi$ to $x = m\pi$. Also, in (24), take $\gamma = 0$, $\theta' = t\theta$, $= 1$. This leads naturally to

$$(29) \qquad g(t, m) = \frac{1}{m\pi}\int_{-m\pi}^{m\pi} \cos x \cdot \cos tx \, dx$$

where the coefficient $1/m\pi$ is chosen to make $g(1, m) = 1$. With the aid of (2), it is easily seen that

$$(30) \qquad g(t, m) = \frac{2t \sin m\pi t}{m\pi(t^2 - 1)}, \qquad t \neq 1$$

for a given m, the plot of $g(t, m)$ as a function of t consists of a crest above the interval from $t = 1 - 1/m$ to $t = 1 + 1/m$, flanked on each side by depressions only about one-fourth or one-fifth as great in size or amplitude followed by waves of still smaller size—a "vibration" *strongly* "damped" on each side of $t = 1$. It has essentially the same characteristics as curves frequently occurring in periodogram analysis.[1] Only the interval from $t = 1 - 1/m$ to $1 + 1/m$ has in general much significance. Sometimes the two adjacent waves[2] need a little attention. But as $\theta' = t\theta$, the above interval is described by

$$(31) \qquad 1 - \frac{1}{m} < \frac{k}{k'} < 1 + \frac{1}{m},$$

which is another way of writing (28).

As an illustration, suppose that 4 cycles of 12 terms each of cos $(r\,30^\circ + \alpha)$ are used in a test with a significantly large result. Here $k = 12$, $m = 4$. Then (28) would recommend to our consideration periods between 9.6 and 16. Perhaps only those between 10 and 15 would deserve serious attention. Since at points $t = 1 \pm 3/4m$, the curve (30) is less than half as high as at $t = 1$. Another interesting form[3] of (28) is

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1 Rietz-Handbook Loc. cit. p. 172, Figure 17.

2 Schuster, *Terrestrial Magnetism*, Vol. 3 (1898), p. 30.

3 Cf. the Schuster criterion, Rietz Loc. cit. p. 173; Schuster, Loc. cit., p. 30.

$$(32) \qquad \left| k' - k \right| < \frac{k'}{m}$$

Before leaving (30), it may be well to note that $g(t, m)$ does *not* take its maximum exactly at $t = 1$; but at

$$(33) \qquad t = 1 + \frac{3}{3 + 2m^2\pi^2}$$

as may be seen by setting $t = 1 + \tau$ in (30), expanding sin $m\pi t =$ sin $m\pi\tau$ in powers of τ, and setting the first derivative equal to zero. When $m = 1$, $t = 1.13$; when $m = 2$, $t = 1.04$; when m is moderately large, t is very close to 1. In all cases, however, the test function which yields the largest result, when applied to a cosine function with period k' is not that one which exactly fits, but one with period $k = k't$, where in the ideal case represented by (29) this value of t is given by (33). Inasmuch as $t > 1$, there is some *danger*, then, of *overestimating the size of the unknown period* k', if the attempt is made to get a close approximation to k' by using several test periods k in the immediate vicinity of k', and selecting the k giving the maximum result. This is not due to the fact that $G(n)$ in (24) is a *linear* function of the Z_r's. For, if in (29), we should change cos x to sin x, to get the mate of $g(t, m)$, this mate would be zero. Thus, the usual *quadratic* function would reduce to the square of $g(t, m)$, and would have its maximum at the same place given by (33). If the main purpose of an investigation is merely to locate with fair precision those periods whose existence have high probabilities, it may not be necessary to refer to (33).

Going back to the constituents of W_r in (16) we see that if n terms of $\sum W_r \cos(r\theta + \alpha)$ are taken, the Y contribution to this sum increases directly as n; the X contribution, being of chance origin, increases usually about as the square root of n; while the Z contribution oscillates about zero.

V. Convenient Forms for Test Functions

The main points of the theory needed for testing data for periods with the aid of linear functions have now been set forth. In the first place, it appears impossible to demonstrate a periodicity. At best, we

can merely make certain suppositions appear more or less probable or improbable. The method outlined here starts with the assumption that the *order* of the sizes exhibited in the data is a *chance* arrangement. Certain functions are to be computed which under chance conditions would ordinarily keep generally within a certain range. If these functions show no marked tendency to jump the bounds, then the tests yield no positive evidence of periodicity. On the other hand, if these functions take on extremely high values, it appears reasonable to relinquish the supposition that the order of sizes is a chance arrangement and to suppose, rather, that such a periodicity exists as would naturally make the function large. If the data have as a constituent a cosine fluctuation and this is matched by a test cosine curve of the same period and phase, it is easy to see that the sum of products all positive obtained from similarly placed ordinates may be abnormally large. Any k which gives these large results is to be regarded as approximating a probable period.

That the tests may all be conducted in a systematic and uniform manner, some further properties and details may well be noted.

In the first place, only values of $k \geqq 2$ need be considered if the data are regarded as representing a sequence of discrete values, corresponding to values of the time (or other argument) spaced at unit intervals. For suppose that p/q, the period of $\cos\left[2\pi rq/p\right]$, is less than 2. Then for each integer r, $\cos\left[2\pi rq/p\right] = \cos\left[2\pi r(p-q)/p\right]$, the latter with period $p/(p-q) > 2$. This applies, indeed, to the case where the discrete values are integrated values. In fact, since

$$(34) \qquad \int_r^{r+1} \cos\left(\frac{2\pi t}{k} + \beta\right) dt = A\cos\left(\frac{2\pi r}{k} + \beta'\right),$$

where $A = (k/\pi)\sin \pi/k$, $\beta' = (\pi/k) + \beta$, it follows that if there is growth or deposit of $K + \cos\left[2\pi t/k + \beta\right] dt$ in time dt —thus, with period k —then the total deposits in time intervals 0 to 1, 1 to 2, 2 to 3, etc., form a sequence with the same period k.

In the second place, it should be noticed that the case of $k = 2$ is peculiar. In place of (12), we have

$$(35) \qquad \sigma_2^2 = 2\sigma^2 \cos^2\alpha$$

as follows directly from the fact that when $k=2$, $\theta=180^\circ$, and $\cos(180^\circ+\alpha) = -\cos\alpha$. With the phase α small, we have approximately $\sigma_2 = \sigma/\sqrt{2}$.

Let us now suppose that the data in given order are divided into sets of convenient size—say sets of 120 measurements. Let the arithmetic mean and standard deviation of each set be found. If these quantities—in particular, the standard deviation—show violent fluctuations as we pass from one set to the next set, it may be necessary to handle the material in different sets. But suppose these fluctuations appear to keep within reasonable bounds.

In (6), the data were represented by X_r. Later, in order to emphasize the possibility of different constituents, W_r was used in (16). But, for simplicity, let us now return to X_r as a symbol for the rth element of the data. In the first tests, let the period k be a whole number. Moreover, in place of the functions (21), (22) let us introduce the following, for $k>2$.

$$(36) \qquad u = u_j(k) = \frac{1}{\sigma}\sqrt{\frac{2}{k}} \sum_{jk-k}^{jk-1} X_r \sin r\theta, \quad j = 1, 2, 3, \cdots$$

$$(37) \qquad v = v_j(k) - \frac{1}{\sigma}\sqrt{\frac{2}{k}} \sum X_r \cos r\theta, \quad k\theta = 360^\circ$$

$$(38) \qquad u' = u'_j(k) - \frac{1}{\sigma}\sqrt{\frac{2}{k}} \sum X_r \sin(r\theta + 45^\circ)$$

$$(39) \qquad v' = v'_j(k) = \frac{1}{\sigma}\sqrt{\frac{2}{k}} \sum X_r \cos(r\theta + 45^\circ)$$

In the case of $k=2$, replace the radical by $1/\sqrt{2}$. If tests for fractional k are desirable, replace k in (36) to (39) by n, where $n = mk$, m and n whole numbers as in (21).

Here for each individual set—say of 120 measurements—it is assumed that each measurement has the same "expected value" or "Probable value," approximated by the arithmetic mean, and the same "mean error," approximated by the standard deviation σ. In this case, u, v, u', and v' all have the same expected value, zero, by (5), noting that the distributive law holds for expected values. Moreover,

when $k > 2$—see (11), (12), (14), (15)—the mean error of u, v, u', and v' is in each case unity. This is also true when $k = 2$, if the phase has been properly matched—see (35).

To make the tests, then, the functions u, v, u', and v' are computed for certain values of k —perhaps for the sub-multiples 2, 3, 4, 5, 6, 8, . . . of 120. In this way, for $k = 6$, twenty values would be found for each of the four functions. The information thus found may not be very significant. But if not, we may combine results as follows. Let $s = q^2$, where q is a whole number. Let

$$(40) \qquad U_1 = \frac{1}{q}\,(u_1 + u_2 + \cdots + u_s) \;; \quad U_2 = \frac{1}{q}\,(u_{s+1} + \cdots + u_{2s}),$$

etc., and form similar expressions for $V_1, V_2, \ldots \; U_1', U_2', \ldots \; V_1', V_2', \ldots$ Each of these functions has expected value *zero* and mean error *unity*. To illustrate—suppose that $u_1(6) = 1.8$; $u_2(6) = 2.1$; $u_3(6) = 1.7$; $u_4(6) = 1.8$. These results taken individually would not furnish strong evidence for a period of 6. Some statisticians regard a variation equal to three times the "probable error" or two times the standard deviation as "significant"—in which case (6) 2.1 would be significant. But such evidence is not overwhelming. But, by (40), $U_1 = 3.7$. Here U_1, with mean value zero, has jumped up to an absolute value 3.7 times its standard deviation, unity. On a pure chance basis, in normal distributions, this would happen only about twice in 10,000 trials, on the average. Altho $U_1 = 3.7$ affords no demonstration of a period of 6, the result is at least highly significant. If such high values occur repeatedly in using $k = 6$, we would be justified in asserting that the data contain a constituent with period somewhere near 6.

Moreover, the process (40) is subject to iteration—as long as the data hold out. If $s' = q'^2$, then $(U_1 + U_2 + \ldots + U_s)/q'$ is a function with mean value zero, and mean error unity.

When the change in standard deviation is fairly gradual from set to set, the values of u_1, u_2 . . . can be computed without interruption, using proper adjustments for those values of u_j whose terms arise partly from two sets, such as u_8 (16).

Such a result as $u_2(6) = 2.1$ would furnish evidence only for the six measurements from which it was computed; and in the light of (28), with $m = 1$, the implication at most would be for some period

greater than 3. But $U_1 = 3.7$ would furnish strong evidence that in the 24 measurements covered there was a constituent with period between 4.8 and 8—taking $m = 4$ in (28).

The technique of computation would present a few problems. In some cases (6) would be utilized. Certain tables[1] of products with the harmonic factors as multiplicands may be of assistance. Or certain tables may be constructed for use with the aid of an adding machine—with complements listed to take the place of negative numbers. Only u and v in (36), (37) need be computed directly; for u' and v' may be found at once—see (23). But it would seem advisable to compute u' or v' as a check. Graphs showing the progress of the functions u, v, etc., may be constructed.

The interpretation of the results would often be difficult because different sections of the data would frequently give different indications. Again, if two layers of rock are counted as one, an error would be introduced. But this would affect the u_j, v_j, ... involved, not the preceding or following u_j, v_j. Indeed, if an actual period is present, as indicated by the u's, an error of merging may merely shift the "burden of proof" to one of the other functions v, u', or v'. Certain cyclic changes, bringing u, u', v, v' into prominence in rotation, may indicate that the test period k is close to an actual period but with a discrepancy large enough to produce a systematic advance of phase. Many similar principles commonly employed in period testing could be used to advantage in the method here outlined.

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1 E. g., L. W. Pollak. "Rechentafeln zur Harmonischen Analyse."

Edward L. Dodd

SYNOPSIS OF ELEMENTARY MATHEMATICAL STATISTICS[1]

By

B. L. Shook

Section IV. The Graphical Representation of Frequency Distributions

25. The investigation of a frequency distribution is greatly facilitated by presenting the data graphically by means of either a *Frequency Polygon* or a *Histogram*, depending upon the nature of the distribution.

For a distribution of discrete variates the frequencies are represented by ordinates whose lengths are proportional to the various frequencies and whose abscissae correspond to the variates of the distribution. The shape of the distribution is rendered more apparent by either connecting the tops of the ordinates by straight lines, thus forming a *Frequency Polygon*, or drawing a *Frequency Curve* that approximately passes through the vertices of the polygon. Figure I presents the Frequency Polygon derived from the data of Table XI. In addition a curve has been drawn to illustrate the general trend of the distribution.

If the frequency distribution under examination be one of grouped discrete or continuous variates it will be found that the *Histogram* is best suited for graphical representation. A Histogram is a series of rectangles erected on bases that are proportional to the class intervals and with altitudes proportional to the respective class frequencies. Thus, in this case, the frequencies are represented by areas. The shape of the distribution may be emphasized by constructing a continuous fre-

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1 A continuationn of an article bearing the same caption appearing in Vol. I, No. 1, of the Annals.

quency curve such that the areas under the curve between the ordinates at the lower and upper boundaries of the various rectangles should equal approximately the areas of the corresponding rectangles. Two examples are presented, both the distributions are composed of continuous variates, one exhibiting positive skewness and the second negative. The numerical data and corresponding Histograms are presented in Tables XII and XIII and Figures II and III respectively.

TABLE XI

Distribution of Frequency of glands in the right fore-leg of 2,000 female swine[1]

v	fv	t	ft
0	15	-2.083	.013
1	209	-1.488	.176
2	365	- .893	.307
3	482	- .298	.405
4	414	.297	.348
5	277	.892	.233
6	134	1.487	.113
7	72	2.082	.061
8	22	2.677	.018
9	8	3.272	.007
10	2	3.867	.002

$M_v = 3.501$ $N = 2000$

$\sigma_v = 1.68077$ $\frac{1}{\sigma} = .594965$

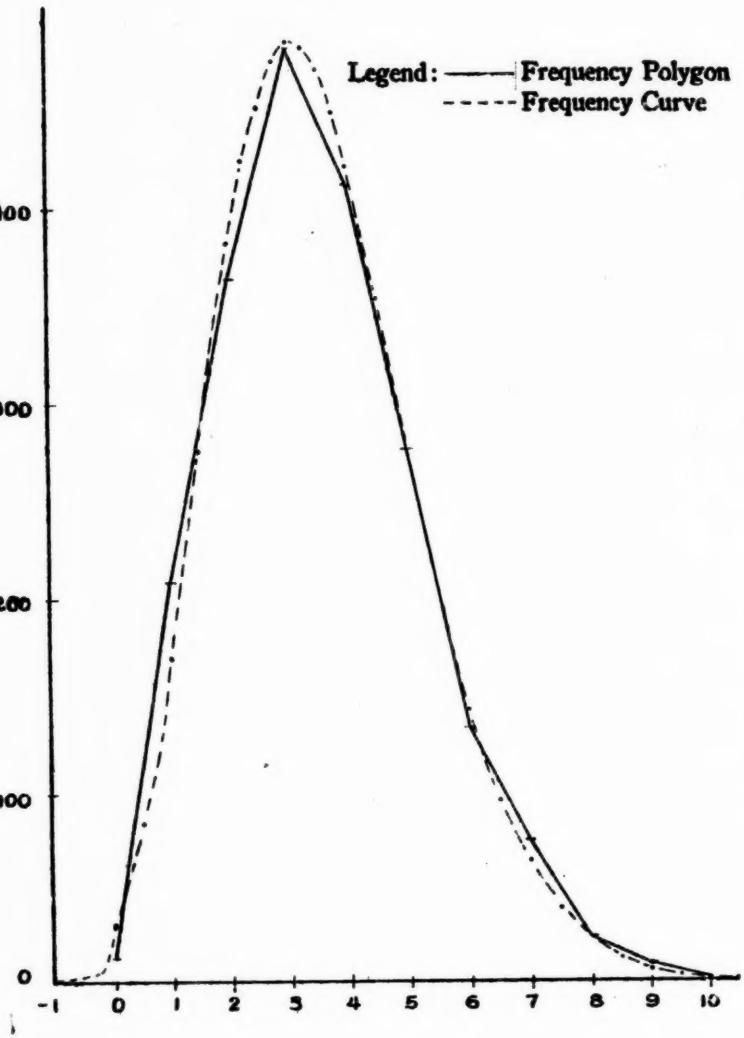
$\alpha_{3:v} = .508462$ $\frac{\sigma}{N} = .000840385$

26. It has previously been stated that the three fundamental statistical functions are the Mean, Standard Deviation, and Skewness. The Mean has been defined as a convenient average, and the Standard

1 Davenport, "Statistical Methods," page 35.

FIGURE I

Frequency Distribution of glands in the right fore-leg of 2,000 female swine



Deviation measures the concentration of the variates about this average. Skewness has not, however, been so clearly explained. If the variates of a distribution be symmetrically arranged about their mean, then $\mu_{3:v}$ or the third moment about the mean will be zero. Under these conditions $\alpha_{3:v}$, or the coefficient of skewness, must also be zero. Thus $\alpha_{3:v}$ measures the degree to which a frequency distribution is symmetrical. If $\alpha_{3:v}$ is zero, then from the standpoint

TABLE XII

Weights of White Boys - 30 to 33 months
(Correct to nearest pound)

Class Mark	f	t	ft
21	3	-2.90	.009
22	3	-2.50	.009
23	11	-2.09	.032
24	27	-1.69	.079
25	65	-1.28	.191
26	101	- .88	.297
27	135	- .47	.397
28	136	- .07	.400
29	128	.34	.376
30	105	.75	.309
31	59	1.15	.173
32	30	1.56	.088
33	15	1.96	.044
34	7	2.37	.021
35	5	2.77	.015
36	8	3.18	.024
37	1	3.58	.003
38	1	3.99	.003

$M_v = 28.16190$ $\qquad N = 840$

$\sigma_v = 2.46837$ $\qquad \frac{1}{\sigma} = .405126$

$\alpha_{3:v} = .427969$ $\qquad \frac{\sigma}{N} = .00293854$

of the present synopsis the distribution may be considered normal, for if such a distribution be graphed in standard units it will follow the locus of the well known Normal Curve of Error. Accordingly it would seem logical to expect that for each value of α_3 there is *one* standard curve which is the locus toward which all distributions with that degree

TABLE XIII

Barometric Heights for Daily Observations During Thirteen Years at Llandudno, England[1]

(Original measurements to nearest millimeter)

Class Mark	t	f	ft
28.35	–4.38	1	.001
28.55	–3.82	2	.001
28.75	–3.26	8	.005
28.95	–2.71	30	.018
29.15	–2.15	74	.045
29.35	–1.59	166	.102
29.55	–1.04	368	.226
29.75	– .48	509	.313
29.95	.08	656	.403
30.15	.63	580	.356
30.35	1.19	353	.217
30.55	1.75	140	.086
30.75	2.31	30	.018
30.95	2.86	5	.003

$M_v = 29.9221$ $\quad N = 2922$

$\sigma_v = .359014$ $\quad \frac{1}{\sigma} = 2.78541$

$\alpha_{3:v} = -.32919$ $\quad \frac{\sigma}{N} = .000614329$[2]

1 Karl Pearson and A. Lee, "Philosophic Transactions," p. 428 (1897).

2 This formula assumes that the class interval is unity, the proper value of $\frac{\sigma}{N}$ is therefore 5 times the value as ordinarily computed.

FIGURE II

Weights of White Boys (30 to 33 months)

Histogram and Frequency Curve

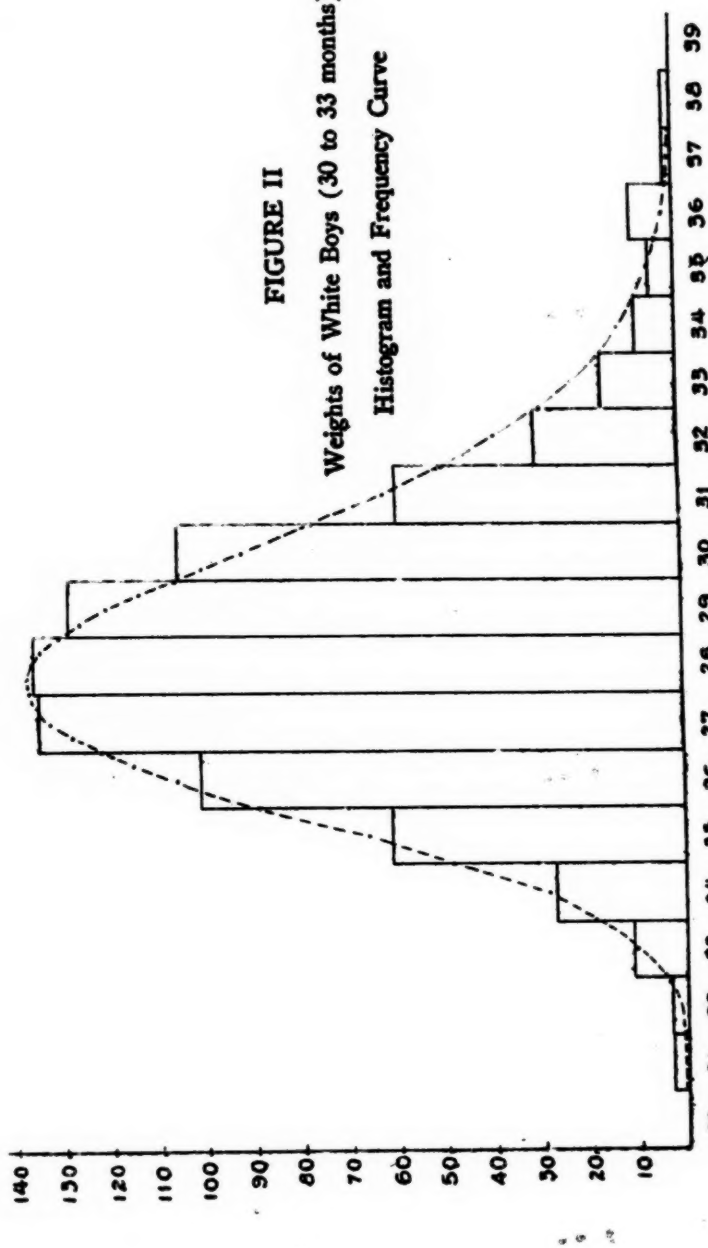
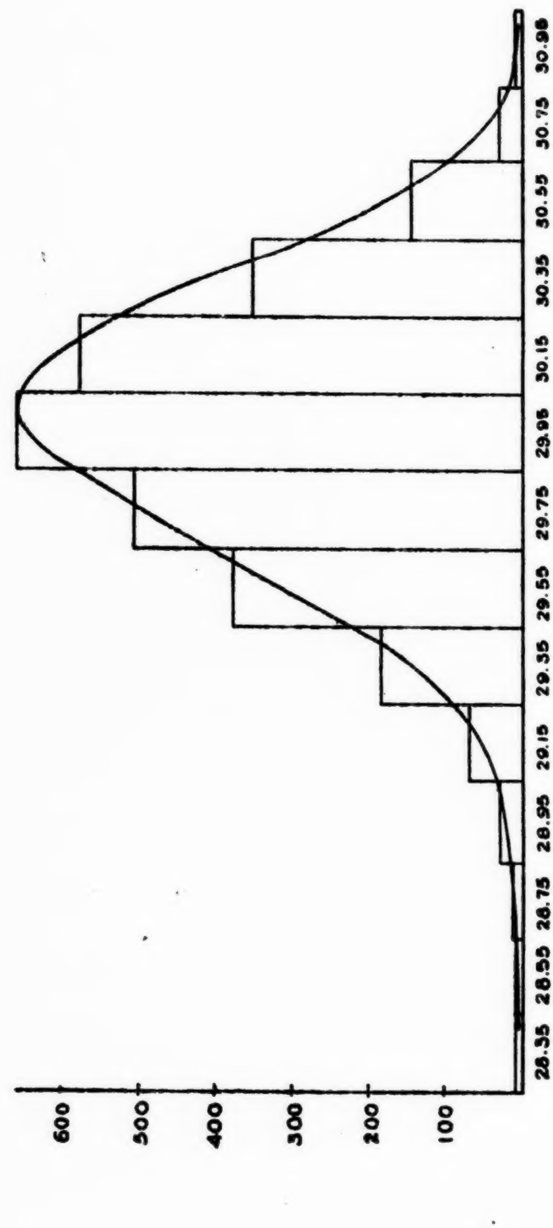


FIGURE III

Barometric Heights Recorded Daily at Llandudno, England



of skewness approach. The one essential is that the unit of measurement must be removed from the data, that is each distribution should be expressed in terms of the standard variates t and the corresponding frequencies f_t . As before, the standard variate t_i corresponding to v_i is obtained from the following formula:

$$t_i = \frac{v_i - M_v}{\sigma_v} = \frac{\bar{v}_i}{\sigma_v}$$

Similarly, the frequencies for each of the standard variates is defined as follows:

$$f_t = \frac{\sigma_v}{N} f_v \tag{26}$$

These two formulae will enable one to analyze all distributions entirely independent of the unit involved. In Figure IV the three distributions graphically presented in Figures I, II and III are shown contrasted with the Normal Curve. The numerical values of t and f_t for each distribution are given in the corresponding table. The values may be obtained in each case by employing the continuous process described in Section I. It will be noticed that the two distributions with positive skewness of .5 and .4 respectively reach their maximum in advance of the Normal Curve and approach the zero limit more gradually for positive values of the standard variates. Accordingly, for the distribution exhibiting negative skewness, the positions are reversed and the more gradual approach to the zero limit occurs for the negative values of the standard variates. In general, a distribution having skewness within the limits ± .3 will exhibit very little deviation from the normal curve when presented graphically in this manner.

Summary of Section IV—

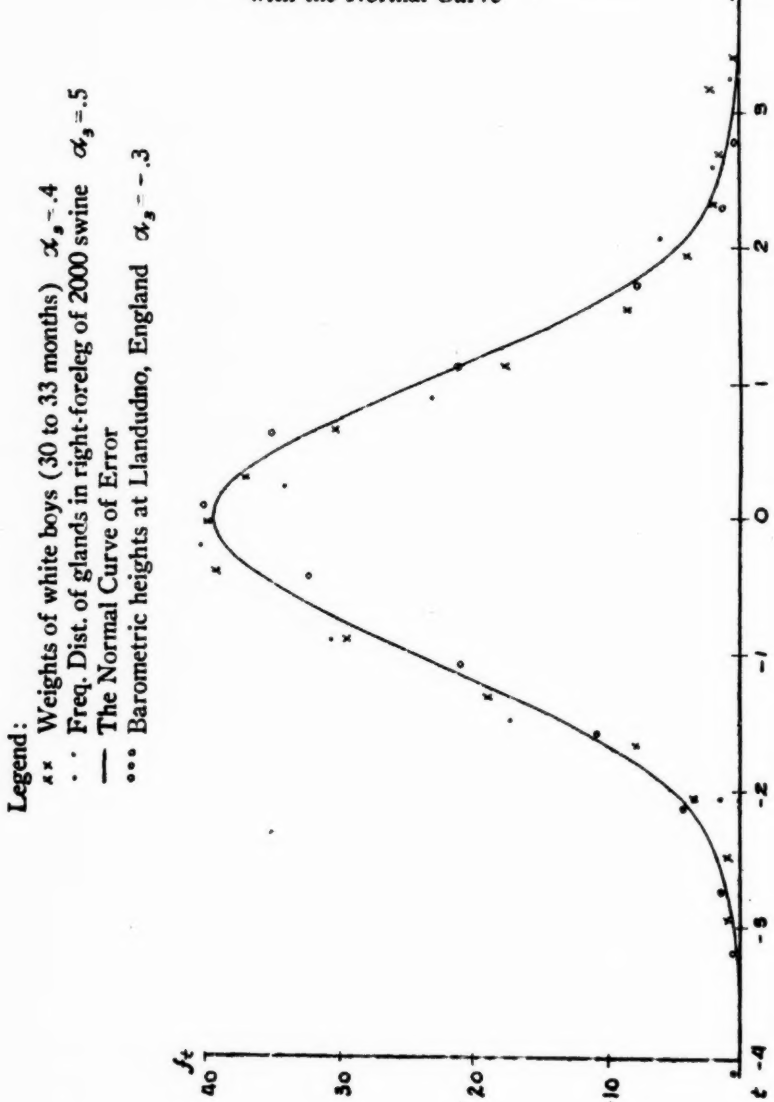
It is usually found very advantageous in the investigation of frequency distributions to present the data graphically. A distribution of discrete variates should be represented by a Frequency Polygon and one of continuous variates by a Histogram. In either case a free hand curve may be drawn indicating the general trend of the distribution and is called the Frequency Curve. The *Standardized* Curve is obtained by plotting the variates and their corresponding frequencies in *standard* form by means of the following formulae:

$$t_i = \frac{v_i - M_v}{\sigma_v} = \frac{\bar{v}_i}{\sigma_v}$$

$$f_t = \frac{\sigma_v}{N} \cdot f_v \,.$$

FIGURE IV

The Graphical Comparison of Three Distributions
with the Normal Curve



Section V. The Inverse Problem

27. From the standpoint of Elementary Mathematical Statistics we may say that the Mean, Standard Deviation, and Skewness together with its total frequency completely characterize a distribution. If this statement were accurate it would be possible to reproduce any distribution if its three elementary functions and total frequency were known. A tabulation of Pearson's Type III Curves for various degrees of skewness affords, for the purposes of Elementary Statistics, the most satisfactory representation of frequency distributions from the point of view of both effectiveness and facility in using[1]. In order to illustrate the method several numerical examples are included. In Table XIV the illustration is one of discrete variates.

1 L. R. Salvosa, "Tables of Pearson's Type III Function," The Annals of Mathematical Statistics, May, 1930.

TABLE XIV

Frequency Distribution of Number of Glands in the Right Foreleg of 2,000 Female Swine

v (1)	t (2)	f_t (3)	Predicted Frequency (4)	Observed Frequency (5)
0	−2.08	.026952	32	15
1	−1.49	.141661	169	209
2	− .89	.320068	381	365
3	− .30	.409193	487	482
4	.30	.353689	421	414
5	.89	.229770	274[1]	277
6	1.49	.118287	141	134
7	2.08	.051638	62[1]	72
8	2.68	.019220	23	22
9	3.27	.006459	8	8
10	3.87	.001925	2	2
Total			2000	2000

$M = 3.501$ $\quad N = 2000$

$\sigma = 1.68077$ $\quad \frac{1}{\sigma} = .594965$

$\alpha_3 = .508462$ $\quad \frac{N}{\sigma} = 1189.93$

Explanation. In every case the value of α_3 is taken to the nearest tenth and the value of t to the nearest hundredth. In the examples included no interpolation has been made for any value.

Columns (1) and (2) of Table XIV contain the variates and the corresponding values of t obtained by means of the continuous process. Column (3) is obtained directly from the Table of Ordinates of the Pearson Type III Function. All values may be found in the

1 In order to obtain $N = 2000$ it was necessary to increase these frequencies by 1, although the fractional value was slightly less than than the necessary .5.

column with skewness = .5 and opposite the respective value of t. Since these are the Standard Frequencies f_t, the predicted frequencies for each variate may be obtained from the following formula.

$$\because f_t = \frac{\sigma}{N} \cdot f_v$$

(27) $$\therefore f_v = \frac{N}{\sigma} \cdot f_t$$

The predicted frequencies in column (4), therefore, are obtained by multiplying column (3) by the value 1189.93. These values are the *graduated* frequencies. The actual observed frequencies are given in column (5).

TABLE XV

Distribution of Weights of White Boys - 30 to 33 Months

(Measurements correct to nearest pound)

Lower Limit of Class (1)	t at Lower Limit (2)	Accumulative Percent Frequency (3)	Percent Frequency (4)	Predicted Frequency (5)	Observed Frequency (6)
20.5	−3.10	.000021	.000357	0	3
21.5	−2.70	.000378	.002990	3	3
22.5	−2.29	.003368	.013174	11	11
23.5	−1.89	.016542	.039440	33	27
24.5	−1.48	.055982	.079399	67	65
25.5	−1,08	.135381	.127331	107	101
26.5	− .67	.262712	.154908	130	135
27.5	− .27	.417620	.163707	138	136
28.5	.14	.581327	.140357	118	128
29.5	.54	.721684	.110157	93	105
30.5	.95	.831841	.073061	61	59
31.5	1.35	.904902	.045897	39	30
32.5	1.76	.950799	.025019	21	15
33.5	2.16	.975818	.013225	11	7
34.5	2.57	.989043	.006176	5	5
35.5	2.97	.995219	.002845	2	8
36.5	3.38	.998064	.001172	1	1
37.5	3.78	.999236	.000483	0	1
38.5	4.19	.999719	.000218	0	0
Total				840	840

$M = 28.16190$ $N = 840$

$\sigma = 2.46837$ $\frac{1}{\sigma} = .404126$

$\alpha_3 = .427969$

Explanation:

28. Since Table XV is a distribution of continuous variates, it is necessary to use the Table of Areas of the Pearson Type III Curve. The values in this table are the *accumulated percent* of the standard curve *below* a specified value of t. The method of prediction is therefore to estimate the per cent of the distribution lying *between* the consecutive lower limits of each class. In column (1) of Table XV are given the lower limit of each class and in Column (2) the value of t at this lower limit. Column (3) is taken directly from the Table of Areas of the Pearson Type III Function, $\alpha_3 = .4$, and represent the percent of the distribution lying *below* the particular value of t. In order to find the percentage of the distribution in each class, it is merely necessary, therefore, to difference column (3). For example, the first value, .000357, is found by subtracting .000021 from .000378. In order to find the predicted frequencies in column (5), N, or the total frequency, should be multiplied by each value in column (4). The observed frequencies are given in column (6).

TABLE XVI

Barometric Heights for Daily Observations During Thirteen Years at Llandudno, England

(Correct to the nearest millemeter)

Lower Limit of Class (1)	t (2)	Accumulative Percent Frequency (3)	Percent Frequency (4)	Predicted Frequency (5)	Observed Frequency (6)
28.25	-4.66	.999956	.000171	1	1
28.45	-4.10	.999785	.000739	2	2
28.65	-3.54	.999046	.002716	8	8
28.85	-2.99	.996330	.009081	27	30
29.05	-2.43	.987249	.025770	75	74
29.25	-1.87	.961479	.061380	179	166
29.45	-1.31	.900099	.117068	342	368
29.65	- .76	.783031	.185122	541	509
29.85	- .20	.597909	.222074	649	656
30.05	.36	.375835	.192952	564	580
30.25	.91	.182883	.121528	355	353
30.45	1.47	.061355	.048526	142	140
30.65	2.03	.012829	.011285	33	30
30.85	2.58	.001544	.001468	4	5
31.05	3.14	.000076	.000076	0	0
Total				2922	2922

$M = 29.92207$ $\quad N = 2922$

$\sigma = .359014$ $\quad \frac{1}{\sigma} = 2.78541$

$\alpha_3 = -.32919$

Explanation:

29. Although the data of Table XVI is also a distribution of continuous variates, it will be noticed that in this case the coefficient of

skewness is negative. Since the Tables include only positive values of α_3, it seems desirable to explain the procedure for such a distribution. If a frequency curve having pronounced positive skewness be graphed on rather fine paper and then held to the light or in front of a mirror, it will be seen that the distribution will seem to show negative skewness to the same degree in which it formerly displayed positive. This being true, it is possible to use the Tables for all cases of negative skewness by merely changing the sign of t, and if an area is desired it is necessary to reverse the order of differencing. Three examples are given in order to cover as many different cases.

Illustration 1, $\alpha_3 = -.5$, required the percentage of the area of the standardized curve lying between $t = -2.43$ and $t = -1.98$. From the tables under the column for skewness = .5.

$$t = +2.43, \text{ percent of area} = .983883$$

$$t = 1.98, \text{ percent of area} = .964416$$

The percentage lying between these two values of t is therefore $.983883 - .964416 = .019467$.

Illustration 2, if $\alpha_3 = -.8$, required the percentage of the area lying between $t = -.02$ and $t = .25$. Using the Table of Areas in the column for skewness of .8,

$$\text{If } t = +.02, \text{ percent of area} = .561064$$

$$t = -.25, \text{ percent of area} = .450687$$

To find the percent of the area merely subtract as before, $.561064 - .450687 = .110377$.

Illustration 3, if $\alpha_3 = -.2$, required the percentage of the area lying between $t = .52$ and $t = 1.63$. Again referring to the Tables of Areas, we find for $\alpha_3 = .2$

$$\text{If } t = -.52, \text{ percent of area} = .310015$$

$$\text{If } t = -1.63, \text{ percent of area} = .045108$$

Accordingly, the required percentage is $.310015 - .045108 = .264907$.

TABLE XVII

Expansion of $(5/6+1/6)^{180}$

v (1)	t (2)	f_t (3)	Pred. Freq. (4)	Obs. Freq. (5)
14	−3.21	.001468	0	0
15	−3.01	.003013	1	1
16	−2.81	.005919	1	1
17	−2.61	.010954	2	2
18	−2.41	.019227	4	4
19	−2.21	.032053	7	6
20	−2.01	.050807	10	10
21	−1.81	.076658	15	16
22	−1.60	.112095	23	23
23	−1.40	.153377	31	31
24	−1.20	.200401	40	41
25	−1.00	.250281	50	51
26	− .80	.299057	60	61
27	− .60	.342196	69	69
28	− .40	.375301	76	75
29	− .20	.394857	80	79
30	.01	.398640	80	80
31	.21	.386166	78	77
32	.41	.359746	72	72
33	.61	.322535	65	64
34	.81	.278510	56	56
35	1.01	.231792	47	46
36	1.21	.186059	38	37
37	1.41	.144144	29	29
38	1.62	.106201	21	22
39	1.82	.076661	15	16
40	2.02	.053513	11	11
41	2.22	.036145	7	8
42	2.42	.024163	5	5
43	2.62	.014975	3	3
44	2.82	.009196	2	2
45	3.02	.005476	1	1
46	3.23	.003077	1	1
47	3.43	.001723	0	0

$M_v = 29.973$ $\qquad N = 1000$

$\sigma_v = 4.96853$ $\qquad t_i = .201267\,v_i - 6.032569$

$\alpha_{3v} = .108097$ $\qquad \frac{N}{\sigma} = 201.267$

TABLE XVIII

Expansion of $(5/6+1/6)^{180}$

Class (1)	Lower Limit (2)	t (3)	Accumulated Percent Freq. (4)	Percent Freq. (5)	Pred. Freq. (6)	Obs. Freq. (7)
11–	10.5	–3.92	.000014	.000213	0	0
14–	13.5	–3.32	.000227	.002128	2	2
17–	16.5	–2.71	.002355	.012561	13	12
20–	19.5	–2.11	.014916	.049031	49	49
23–	22.5	–1.50	.063947	.120876	121	123
26–	25.5	– .90	.184823	.203066	203	205
29–	28.5	– .30	.387889	.239543	239	236
32–	31.5	.31	.627432	.191988	192	192
35–	34.5	.91	.819420	.112488	112	112
38–	37.5	1.51	.931908	.048675	49	49
41–	40.5	2.12	.980583	.015048	15	16
44–	43.5	2.72	.995631	.003627	4	4
47–	46.5	3.33	.999258	.000742	1	0

$M_y = 29.973$ $\qquad N = 1000$

$\sigma_y = 4.96848$ $\qquad \frac{1}{\sigma} = .201269$

$\alpha_{3,y} = .105899$

30. As further numerical examples the three illustrated problems used in Section III have been graduated. The complete numerical solution will be found in Tables XVII, XVIII and XIX.

Summary of Section V—

Knowing the three fundamental functions and the total frequency of a distribution, it is possible to obtain predicted or graduated frequencies for that distribution with a surprising degree of accuracy. This is accomplished through the use of tables of the standard ordinates and accumulated percentage areas of the Pearson Type III Curves.

TABLE XIX

Weights of 1000 Female Students

(Original measurements to nearest .1 lb.)

Class (1)	Lower Limit (2)	t (3)	Accumulated Percent Freq. (4)	Percent Freq. (5)	Pred. Freq. (6)	Obs. Freq. (7)
70–	69.95	–2.88	.000000	.000000	0	2
80–	79.95	–2.29	.000000	.003358	4	16
90–	89.95	–1.70	.003358	.102159	102	82
100–	99.95	–1.11	.105517	.238290	238	231
110–	109.95	– .52	.343807	.249585	250	248
120–	119.95	.07	.593392	.183665	184	196
130–	129.95	.66	.777057	.111093	111	122
140–	139.95	1.25	.888150	.059338	59	63
150–	149.95	1.84	.947488	.029412	29	23
160–	159.95	2.44	.976900	.013209	13	5
170–	169.95	3.03	.990109	.005791	6	7
180–	179.95	3.62	.995900	.002445	3	1
190–	189.95	4.21	.998345	.001002	1	2
200–	199.95	4.80	.999347	.000400	0	1
210–	209.95	5.39	.999747	.000157	0	1
220–	219.95	5.98	.999904	.000096	0	0

$M_v = 118.74$ $\quad$ $N = 1000$

$\sigma_v = 16.9175$ $\quad$ $\frac{1}{\sigma} = .0591104$

$\alpha_{3:v} = .976424$

It should be remembered that in advanced statistics moments higher than the third are necessary to characterize a distribution, but from the elementary viewpoint, the Mean, Standard Deviation and Skewness are considered to completely characterize a distribution.

Section VI. Bernoulli's Theorem

31. *Factorials.* For convenience, the product of the first n consecutive integers is called "factorial n" and is designated by the symbol $\underline{|n}$. Thus

$$\underline{|3} = 1\cdot 2\cdot 3 = 6,\quad \underline{|5} = 1\cdot 2\cdot 3\cdot 4\cdot 5 = 120,\quad \frac{\underline{|8}}{\underline{|5}} = 8\cdot 7\cdot 6 = 336.$$

Combinations. The number of combinations, each of r things, that can be formed from n things, is represented by the symbol ${}_nC_r$. Texts on elementary algebra show that

$$(28)\qquad {}_nC_r = \frac{\underline{|n}}{\underline{|r}\,\underline{|n-r}} = \frac{n(n-1)(n-2)\cdots(n-\overline{r-1})}{1\cdot 2\cdot 3\cdots r}$$

For example, suppose we desire to find the number of different committees, each of three persons, that can be selected from five individuals. If we designate the five individuals by the letters A, B, C, D and E, we observe that committees of three may be systematically enumerated as follows:

ABC, ABD, ABE, ACD, ACE, ADE, BCD, BCE, BDE, CDE

The number of committees, which we just enumerated as 10, agrees with the value found by formula (28), for since here $n=5$, $r=3$,

$${}_nC_r = {}_5C_3 = \frac{\underline{|5}}{\underline{|3}\cdot\underline{|2}} = \frac{5\cdot 4}{1\cdot 2} = 10$$

Another illustration: The number of different committees, each composed of seven individuals, that can be selected from ten candidates is

$${}_{10}C_7 = \frac{\underline{|10}}{\underline{|7}\,\underline{|3}} = \frac{10\cdot 9\cdot 8}{1\cdot 2\cdot 3} = 120$$

and the number of combinations, each of three, that can be formed from ten items is

$${}_{10}C_3 = \frac{\underline{|10}}{\underline{|3}\ \underline{|7}} = 120$$

It should be noted that ${}_{10}C_7 = {}_{10}C_3$, and in general that

$$(29) \qquad {}_nC_r = {}_nC_{n-r}$$

This follows from the fact that the number of ways of selecting r items from n is equal to the number of ways of rejecting $(n-r)$ from n. Thus, every time three are selected from ten, seven are rejected. Therefore the number of ways of selecting three from ten, ${}_{10}C_3$, is also equal to ${}_{10}C_7$.

We shall have occasion to refer to the following tabulation of values of ${}_nC_r$.

TABLE XX

Values of ${}_nC_r$

N	r												
	0	1	2	3	4	5	6	7	8	9	10	11	12
1	1	1											
2	1	2											
3	1	3	3	1									
4	1	4	6	4	1								
5	1	5	10	10	5	1							
6	1	6	15	20	15	6	1						
7	1	7	21	35	35	21	7	1					
8	1	8	28	56	70	56	28	8	1				
9	1	9	36	84	126	126	84	36	9	1			
10	1	10	45	120	210	252	210	120	45	10	1		
11	1	11	55	165	330	462	462	330	165	55	11	1	
12	1	12	66	220	495	792	924	792	495	220	66	12	1

32 *Binomial Theorem.* By repeated multiplication we find that

$$(a+b)^1 = a+b$$

$$(a+b)^2 = a^2+2ab+b^2$$

$$(a+b)^3 = a^3+3a^2b+3ab^2+b^3$$

$$(a+b)^4 = a^4+4a^3b+6a^2b^2+4ab^3+b^4$$

etc.

By mathematical induction it can be shown that for positive integer values of n

$$(30)\quad (a+b)^n = a^n + na^{n-1}b + \frac{n(n-1)}{1\cdot 2}a^{n-2}b^2 + \frac{n(n-1)(n-2)}{1\cdot 2\cdot 3}a^{n-3}b^3 + \cdots\cdots$$

This equation is known as the binomial theorem and may be written more compactly, if n is an integer, in the following form:

$$(31)\quad (a+b)^n = a^n + {}_nC_1 a^{n-1}b + {}_nC_2 a^{n-2}b^2 + {}_nC_3 a^{n-3}b^3 + \cdots\cdots$$

Using Table XX, we may write down at once that

$$(a+b)^{12} = a^{12} + 12a^{11}b + 66a^{10}b^2 + 220a^9b^3 + \cdots\cdots + 66a^2b^{10} + 12ab^{11} + b^{12}$$

Bernoulli's Series. If p denote the probability that an event will happen in a single trial, and q the probability that it will not happen in that trial, $p+q=1$, then the probability that the event will happen exactly 0, 1, 2, . . . x times during r trials is given by the respective terms of the binomial expansion

$$(32)\quad (q+p)^r = q^r + {}_rC_1 q^{r-1}p + {}_rC_2 q^{r-2}p^2 + \cdots {}_rC_x q^{r-x}p^x + \cdots\cdots$$

To illustrate. If a coin be tossed, we may assume *a priori* that the probability that heads will turn up is $p=1/2$ and the probability that heads will not turn up is $q=1/2$. If an individual tosses the coin twelve times in succession, it is possible that heads may turn up on no occasion, or heads may turn up exactly 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 or 12 times, respectively. By formula (32), these chances are equal respectively to the successive terms of the expansion of $\left(\frac{1}{2}+\frac{1}{2}\right)^{12}$, namely

$$\left(\frac{1}{2}\right)^{12}+{}_{12}C_1\left(\frac{1}{2}\right)^{11}\left(\frac{1}{2}\right)+{}_{12}C_2\,\frac{1}{2}^{10}\left(\frac{1}{2}\right)^{2}+\cdots\cdots+{}_{12}C_{11}\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^{11}+{}_{12}C_{12}\left(\frac{1}{2}\right)^{12}$$

Denoting the probability that heads will turn up on exactly x occasions by P_x, and referring to Table XX for values of ${}_{12}C_x$, we have that

$$P_{12}=\frac{1}{4096},\quad P_{11}=\frac{12}{4096},\quad P_{10}=\frac{66}{4096},\quad P_9=\frac{220}{4096}$$

TABLE XXI

Values of the Terms in the expansion of $\left(\frac{1}{2}+\frac{1}{2}\right)^{12}$

Number of Successes	$r=12$, $q=.5$, $p=.5$		Observed Frequencies
	Probability P_x	Expected Freq. $4096\,P_x$	
(1)	(2)	(3)	(4)
0	1/4096	1	0
1	12/4096	12	7
2	66/4096	66	60
3	220/4096	220	198
4	495/4096	495	430
5	792/4096	792	731
6	924/4096	924	948
7	792/4096	792	847
8	495/4096	495	536
9	220/4096	220	257
10	66/4096	66	71
11	12/4096	12	11
12	1/4096	1	0
Total	1	4096	4096

33. *Expectation.* If p denote the probability of success for each of n trials, then pn is defined as the expected number of successes in n trials. For example, we have just shown that the *a priori* probability of throwing heads twelve successive times with a coin is equal to $P_{12} = \frac{1}{4096}$. Therefore if twelve coins be tossed simultaneously on 4096 occasions, we expect that all twelve coins will turn up heads on only one occasion. Likewise, the expected number of times that exactly ten heads and two tails would turn up is equal to $4096 \cdot P_{10} = 66$, and that exactly half of the coins would turn heads only $4096 \cdot P_6 = 924$ times.

It will be seen that the sum of all the probabilities in column (2) is unity. This follows from the fact that these values are the several terms of the expansion of $(q+p)^r$, and since $q+p=1$, therefore $(q+p)^r = 1$.

TABLE XXII

Values of the Terms in the Expansion of $\left(\frac{5}{6}+\frac{1}{6}\right)^{12}$

Number of Successes (1)	$r=12$, $q=5/6$ $p=1/6$ Probability P_x (2)	Expected Freq. (3)	Observed Frequencies (4)
0	.11216	459	447
1	.26918	1103	1145
2	.29609	1213	1181
3	.19739	808	796
4	.08883	364	380
5	.02843	116	115
6	.00663	27	24
7	.00114	5	7
8	.00014	1	1
9	.00001	0	0
10	.00000	0	0
11	.00000	0	0
12	.00000	0	0
Total	1.00000	4096	4096

A second illustration: Suppose twelve dice are thrown and that only a throw of 6 is to be considered a success. By formula (32), therefore, the expansion of

$$\left(\frac{5}{6}+\frac{1}{6}\right)^{12}=\left(\frac{5}{6}\right)^{12}+{}_{12}C_1\left(\frac{5}{6}\right)^{11}\left(\frac{1}{6}\right)+{}_{12}C_2\left(\frac{5}{6}\right)^{10}\left(\frac{1}{6}\right)^{2}+\cdots\cdots$$

are equal respectively to the probabilities that exactly 0, 1, 2, ... successes will be obtained in a single throw of the twelve dice, or what is the same thing, in twelve successive throws with a single die.

In this case the probabilities p_x are expressed as decimals, since the expansion contains values of $(6)^{12}$ in the denominators. Therefore 6^{12} is the smallest value of N that will produce integer expected frequencies.

34 We shall now attack a more important problem. Let us consider a hypothetical group of 100,000 individuals, all of the same age and all exposed to the same hazards of life. Moreover, let us assume that the probability that each individual will die within one year is $p = .008$, or that the probability that any specified individual will survive a year is $q = .992$.

By formula (32), the terms of the expansion of $(q+p)^r$ $(.992+.008)^{100,000}$, namely, $(.992)^{100,000}$ ${}_{100,000}C_1\,(.992)^{99,999}$ $(.008)^1+$ ${}_{100,000}C_2\,(.992)^{99,998}$ $(.008)^2$ . . . . ${}_{100,000}C_x$ $(.992)^{100,000-x}$ $(.008)^x$ . . . . represent the probabilities that exactly 0, 1, 2, . . ., x, . . . individuals will die within the year.

The value of $(.992)^{100,000}$ is very small. Thus $(.992)^{100,000} = \left(\frac{992}{1000}\right)^{100,000}$

$\log 992 = 2.9965117$	$\log 992^{100,000} = 299651.17$
$\log 1000 = 3$	$\log 1000^{100,000} = \underline{300000.00}$
	$\log (.992)^{100,000} = \overline{349}.17$

Therefore $.992^{100,000} = .000,000,000\ .\ .\ .\ 15$, where 15 is preceded by 348 zeros. The probability that all would die $(.008)^{100,000}$ is far less than this value.

The values of P_x in Table XXIII are given to the nearest fourth

decimal place. Thus to six decimal places $P_{690} = .000005$ and $P_0 + P_1 + P_2 + \ldots + P_{690} = \sum_0^{690} P_x = .000035$. These values appear in Table XXIII, therefore, as .0000. An inspection of Table XXIII shows that for our hypothetical population

(a) The chance that exactly 800 will die within a year is .0142
(b) The chance that 800 or less will die within a year is .5094.
(c) The chance that 850 or less will die within a year is .9625.
(d) The chance that at least 750 will die within a year is

$$P_{750} + P_{751} + P_{752} + \ldots + P_{100,000} = 1 - \sum_0^{749} P_x = 1 - .0355 = .9645$$

Obviously the sum of all terms from P_0 to $P_{100,000}$ is equal to unity. It is interesting to note that although q is relatively much greater than p, nevertheless the values of P_x are very symetrically arranged about their mean. For example, the first significant term of P_x is $P_{707} = .0001$, and the last significant term is $P_{896} = .0001$. Thus there are 93 significant terms above and 96 terms below P_{800}. However, there are 707 insignificant terms before P_{707} and 99,104 insignificant terms after P_{896}. We have arbitrarily rejected as insignificant any value less than .0001. Had we taken .0000001 as the limit of significance, we would have found that the limiting significant values of P_x are $P_{668} = P_{944} = .0000001$. Here again the significant ranges above and below the expected P_{800} are almost the same.

In general it may be said that unequal values of q and p when associated with large values of r are reflected in an unequal number of insignificant terms in the upper and lower ranges. The significant terms form a distribution which, to the eye, is rather symmetrical.

35. Let us now retrace a few steps. Theoretically, formula (32) enables one to compute the probability that exactly x individuals out of any population of r will die within a year, provided, of course, q and p are known. Actually, however, such computation is very laborious. Thus, it is not easy to show that

$$P_{850} = {}_{100,000}C_{850} \, (.992)^{99150} (.008)^{850} = .0029354$$

TABLE XXIII

Values of P_x and $\sum_{x=0}^{x} P_x$, $P_x = {}_rC_x\, q^{r-x} p^x$ and $r = 100{,}000$,

$q = .999$, $p = .008$

x	P_x	$\sum_0^x P_x$	x	P_x	$\sum_0^x P_x$	x	P_x	$\sum_0^x P_x$
690	.0000	.0000	730	.0006	.0062	770	.0081	.1474
691	.0000	.0000	731	.0007	.0069	771	.0084	.1558
692	.0000	.0000	732	.0007	.0077	772	.0087	.1645
693	.0000	.0001	733	.0008	.0085	773	.0091	.1736
694	.0000	.0001	734	.0009	.0093	774	.0094	.1830
695	.0000	.0001	735	.0010	.0103	775	.0097	.1926
696	.0000	.0001	736	.0010	.0113	776	.0100	.2026
697	.0000	.0001	737	.0011	.0125	777	.0103	.2128
698	.0000	.0001	738	.0012	.0137	778	.0106	.2234
699	.0000	.0001	739	.0013	.0150	779	.0108	.2342
700	.0000	.0002	740	.0014	.0165	780	.0111	.2454
701	.0000	.0002	741	.0016	.0180	781	.0114	.2568
702	.0000	.0002	742	.0017	.0197	782	.0117	.2684
703	.0000	.0002	743	.0018	.0215	783	.0119	.2803
704	.0000	.0003	744	.0019	.0234	784	.0122	.2925
705	.0000	.0003	745	.0021	.0255	785	.0124	.3049
706	.0000	.0004	746	.0022	.0278	786	.0126	.3175
707	.0001	.0004	747	.0024	.0302	787	.0128	.3303
708	.0001	.0005	748	.0026	.0327	788	.0130	.3433
709	.0001	.0005	749	.0027	.0355	789	.0132	.3565
710	.0001	.0006	750	.0029	.0384	790	.0134	.3699
711	.0001	.0007	751	.0031	.0415	791	.0135	.3835
712	.0001	.0008	752	.0033	.0448	792	.0137	.3971
713	.0001	.0009	753	.0035	.0484	793	.0138	.4109
714	.0001	.0010	754	.0037	.0521	794	.0139	.4248
715	.0001	.0012	755	.0040	.0561	795	.0140	.4388
716	.0001	.0013	756	.0042	.0603	796	.0141	.4528
717	.0002	.0015	757	.0044	.0647	797	.0141	.4669
718	.0002	.0017	758	.0047	.0694	798	.0141	.4811
719	.0002	.0019	759	.0049	.0744	799	.0142	.4952
720	.0002	.0021	760	.0052	.0796	800	.0142	.5094
721	.0003	.0023	761	.0055	.0850	801	.0141	.5235
722	.0003	.0026	762	.0058	.0908	802	.0141	.5377
723	.0003	.0029	763	.0060	.0968	803	.0141	.5517
724	.0003	.0033	764	.0063	.1032	804	.0140	.5657
725	.0004	.0037	765	.0066	.1098	805	.0139	.5796
726	.0004	.0041	766	.0069	.1167	806	.0138	.5934
727	.0005	.0046	767	.0072	.1239	807	.0137	.6070
728	.0005	.0051	768	.0075	.1314	808	.0135	.6206
729	.0006	.0056	769	.0078	.1392	809	.0134	.6340

TABLE XXIII (Continued)

x	P_x	$\sum_0^x P_x$	x	P_x	$\sum_0^x P_x$	x	P_x	$\sum_0^x P_x$
810	.0132	.6472	850	.0029	.9625	890	.0001	.9992
811	.0130	.6602	851	.0028	.9652	891	.0001	.9993
812	.0128	.6731	852	.0026	.9678	892	.0001	.9994
813	.0126	.6857	853	.0024	.9702	893	.0001	.9994
814	.0124	.6981	854	.0023	.9725	894	.0001	.9995
815	.0122	.7103	855	.0021	.9746	895	.0001	.9996
816	.0119	.7222	856	.0020	.9766	896	.0001	.9996
817	.0117	.7339	857	.0019	.9785	897	.0000	.9997
818	.0114	.7454	858	.0017	.9802	898	.0000	.9997
819	.0112	.7565	859	.0016	.9818	899	.0000	.9997
820	.0109	.7674	860	.0015	.9833	900	.0000	.9998
821	.0106	.7781	861	.0014	.9847	901	.0000	.9998
822	.0103	.7884	862	.0013	.9860	902	.0000	.9998
823	.0100	.7984	863	.0012	.9872	903	.0000	.9998
824	.0097	.8082	864	.0011	.9883	904	.0000	.9999
825	.0094	.8176	865	.0010	.9893	905	.0000	.9999
826	.0091	.8267	866	.0009	.9902	906	.0000	.9999
827	.0088	.8356	867	.0009	.9911	907	.0000	.9999
828	.0085	.8441	868	.0008	.9919	908	.0000	.9999
829	.0082	.8524	869	.0007	.9926	909	.0000	.9999
830	.0079	.8603	870	.0007	.9933	910	.0000	.9999
831	.0076	.8680	871	.0006	.9939	911	.0000	.9999
832	.0073	.8753	872	.0006	.9945	912		
833	.0071	.8824	873	.0005	.9950	913		
834	.0068	.8891	874	.0005	.9955	914		
835	.0065	.8956	875	.0004	.9959	915		
836	.0062	.9018	876	.0004	.9963	916		
837	.0059	.9077	877	.0004	.9967	917		
838	.0056	.9134	878	.0003	.9970	918		
839	.0054	.9188	879	.0003	.9973	919		
840	.0051	.9239	880	.0003	.9976	920		
841	.0049	.9288	881	.0002	.9978	921		
842	.0046	.9334	882	.0002	.9981	922		
843	.0044	.9378	883	.0002	.9983	923		
844	.0042	.9419	884	.0002	.9984	924		
845	.0039	.9459	885	.0002	.9986	925		
846	.0037	.9496	886	.0001	.9988	926		
847	.0035	.9531	887	.0001	.9989	927		
848	.0033	.9564	888	.0001	.9990	928		
849	.0031	.9595	889	.0001	.9991	929		

It can be done, provided an extensive table of logarithms are available, by using the so-called Stirling's formula

$$\lfloor n = \sqrt{2\pi}\ n^{n+\frac{1}{2}}\ e^{-n+\frac{1}{12n}-\frac{1}{360n^3}+\cdots}$$

where $\pi = 3.14159\ 26535\ 89793 \ldots.$
$e = 2.71828\ 18284\ 59045$

We shall now proceed to develop a method which will enable us to find approximately the value of any term of the expansion of $(q+p)^r$ and the sum of any number of consecutive terms of this series.

In Section V we made use of the fact that the mean, standard deviation, and skewness may be regarded as satisfactorily describing any distribution. We shall now show that for any distribution whose frequencies are proportional to the terms of the expansion of $(q+p)^r$,

$$(33)\qquad \begin{aligned} M &= rp \\ \sigma &= \sqrt{rp(1-p)} \\ \alpha_3 &= \frac{1-2p}{\sigma} \end{aligned}$$

Thus, for the expected distribution of Table XXI, column (3), since $r = 12$, $p = \frac{1}{2}$, $q = \frac{1}{2}$,

$$\begin{aligned} M &= rp = \frac{12}{2} = 6 \\ \sigma &= \sqrt{12 \cdot \frac{1}{2} \cdot \frac{1}{2}} = \sqrt{3} = 1.732 \\ \alpha_3 &= 0 \end{aligned}$$

Similarly, for the expected distribution of Table XXII, column (3), since $r = 12$, $q = 5/6$, $p = 1/6$,

$$\begin{aligned} M &= \frac{12}{6} = 2 \\ \sigma &= \sqrt{12 \cdot \frac{1}{6} \cdot \frac{5}{6}} = \sqrt{\frac{5}{3}} = 1.291 \\ \alpha_3 &= \frac{1-\frac{1}{3}}{1.291} = .516 \end{aligned}$$

Values for these expected distributions may be calculated from the frequencies ${}_{4096}P_x$ in the usual manner. The results will then be found to agree with those obtained as above by means of formulae (33). Since the P_x column in Table XXI is composed of integers they will agree exactly, but since in Table XXII both the probabilities and expected frequencies are approximations, the values of these functions obtained by the two methods may differ slightly. Theoretically those obtained by employing formulae (33) are the more correct.

If, as before, q denote the probability that each individual will die within a year, and P_x the probability that exactly x out of r individuals will die within one year, then the values of P_0, P_1, P_2, ... are equal to the terms of the expansion of $(q+p)^r$ which are shown in frequency distribution form in Table XXIV.

The total of column (2) is obviously equal to N since the values of f_x are merely the expansion of $N(q+p)^r$. Since $q+p=1$, therefore $(q+p)^r=1$, and hence $\Sigma f_x = N$.

If one takes the common factor Nrp out of every term in column (3) of the previous table, it is noted that the sum of this column may be written

$$N\Sigma x f_x = Nrp\left[q^{r-1} + (r-1)q^{r-2}p + \frac{(r-1)(r-2)}{1\cdot 2}q^{r-3}p^2 + \cdots\cdots\right]$$

But the expression within the bracket is merely the expansion of the binomial $(q+p)^{r-1}$. Hence $\Sigma x f_x = Nrp\,[1] = Nrp$. Likewise the sum of the terms in columns (4) and (5) may be factored as follows:

$$\Sigma x(x-1)f_x = Nr(r-1)p^2\left[q^{r-2} + (r-2)q^{r-3}p + \frac{(r-2)(r-3)}{1\cdot 2}q^{r-4}p^2 + \cdots\right]$$

$$= Nr(r-1)p^2(q+p)^{r-2} = Nr(r-1)p^2$$

$$\Sigma x(x-1)(x-2)f_x = Nr(r-1)(r-2)p^3\left[q^{r-3} + (r-3)q^{r-4}p + \frac{(r-3)(r-4)}{1\cdot 2}q^{r-5}p^2 + \cdots\right]$$

$$= Nr(r-1)(r-2)p^3(q+p)^{r-3} = Nr(r-1)(r-2)p^3$$

TABLE XXIV

x (1)	f_x (2)	$x \cdot f_x$ (3)	$x(x-1)f_x$ (4)	$x(x-1)(x-2)f_x$ (5)
0	Nq^r	0	0	0
1	$Nrq^{r-1}p$	$Nrq^{r-1}p$	0	0
2	$N\frac{r(r-1)}{1\cdot 2}q^{r-2}p^2$	$N\frac{r(r-1)}{1}q^{r-2}p^2$	$Nr(r-1)q^{r-2}p^2$	0
3	$N\frac{r(r-1)(r-2)}{1\cdot 2\cdot 3}q^{r-3}p^3$	$N\frac{r(r-1)(r-2)}{1\cdot 2}q^{r-3}p^3$	$N\frac{r(r-1)(r-2)}{1}q^{r-3}p^3$	$Nr(r-1)(r-2)q^{r-3}p^3$
4	$N\frac{r(r-1)(r-2)(r-3)}{1\cdot 2\cdot 3\cdot 4}q^{r-4}p^4$	$N\frac{r(r-1)(r-2)(r-3)}{1\cdot 2\cdot 3}q^{r-4}p^4$	$N\frac{r(r-1)(r-2)(r-3)}{1\cdot 2}q^{r-4}p^4$	$N\frac{r(r-1)(r-2)(r-3)}{1}q^{r-4}p^4$
	etc.	etc.	etc.	etc.
Total	$N(q+p)^r = N$	$Nrp(q+p)^{r-1} = Nrp$	$Nr(r-1)p^2(q+p)^{r-2} = Nr(r-1)p^2$	$Nr(r-1)(r-2)p^3(q+p)^{r-3} = Nr(r-1)(r-2)p^3$

But we may write

$$x(x-1)f_x = x^2 f_x - x f_x \,.$$

$$\therefore \ \Sigma\, x(x-1) f_x = \Sigma\, x^2 f_x - \Sigma\, x f_x$$

$$x(x-1)(x-2) f_x = x^3 f_x - 3x^2 f_x + 2x f_x \,.$$

$$\therefore \ \Sigma\, x(x-1)(x-2) f_x = \Sigma\, x^3 f_x - 3\Sigma\, x^2 f_x + 2\Sigma\, x f_x$$

So we have

$$\Sigma f_x = N$$

$$\Sigma x f_x = Nrp$$

$$\Sigma\, x(x-1) f_x = \Sigma\, x^2 f_x - \Sigma\, x f_x = Nr(r-1)p^2$$

$$\Sigma\, x(x-1)(x-2) f_x = \Sigma\, x^3 f_x - 3\Sigma\, x^2 f_x + 2\Sigma\, x f_x$$

$$= Nr(r-1)(r-2)p^3$$

Therefore

$$\Sigma\, x^2 f_x = \Sigma\, x f_x + Nr(r-1)p^2 = Nrp + Nr(r-1)p^2$$

$$= Nrp + Nr^2p^2 - Nrp^2$$

$$\Sigma\, x^3 f_x = 3\Sigma\, x^2 f_x - 2\Sigma\, x f_x + Nr(r-1)(r-2)p^3$$

$$= 3N(rp + r^2p^2 - rp^2) - 2Nrp + Nr(r-1)(r-2)p^3$$

$$= Nrp + 3Nr^2p^2 - 3Nrp^2 + Nr^3p^3 - 3Nr^2p^3 + 2Nrp^3$$

Hence

$$M_x = \frac{\Sigma x f_x}{\Sigma f_x} = \frac{Nrp}{N} = rp$$

$$\mu_2' = \frac{\Sigma x^2 f_x}{\Sigma f_x} = rp + r^2p^2 - rp^2$$

$$\mu_3' = \frac{\Sigma x^3 f_x}{\Sigma f_x} = rp + 3r^2p^2 - 3rp^2 + r^3p^3 - 3r^2p^3 + 2rp^3$$

$$\mu_2 = \mu_2' - M_x^2 = rp - rp^2 = rp(1-p)$$

$$\mu_3 = \mu_3' - 3M_x\mu_2' + 2M_x^3 = rp - 3rp^2 + 2rp^3$$

$$= rp(1-3p+2p^2) = rp(1-p)(1-2p)$$

The reductions follow since $(q+p) = 1$.

We have finally, that

$$M = rp$$

$$\sigma = \sqrt{\mu_2} = \sqrt{rp(1-p)}$$

$$\alpha_3 = \frac{\mu_3}{\sigma^3} = \frac{rp(1-p)(1-2p)}{\left(\sqrt{rp(1-p)}\right)^3} = \frac{1-2p}{\sigma}$$

Formulae (33) are therefore established.

The equation $M = rp$ shows that for a Bernoulli series the "mean" value is also the "expected" value, since, from our definition of expectation, the expected number of deaths from a group of r individuals is rp.

36. For the distribution of the values of P_x shown in Table XXIII, since $r = 100{,}000$, $q = .992$, $p = .008$

$$M_x = rp = 800$$

$$\sigma = \sqrt{rp(1-p)} = \sqrt{793.6} = 28.1709$$

$$\frac{1}{\sigma} = .0354976$$

$$\alpha_3 = \frac{1-2p}{\sigma} = \frac{.992-.008}{\sigma} = .984(.0354976) = .03493$$

If as before we let $t = \frac{x-M}{\sigma}$, and designate the ordinates of the standard frequency curves by f_t , we can compute any value of P_x with a reasonable degree of accuracy by the formula

(34) $$P_x = \frac{1}{\sigma} f_t$$

For example: Required the probability that exactly 762 individuals will die within one year in a population of 100,000 for which $p = .008$.

As before, we must first express the number of deaths under consideration in standard units, that is since $M_x = rp = 800$,

$$t = \frac{x-M}{\sigma} = \frac{762-800}{\sigma} = -38(.0354976) = -1.3489$$

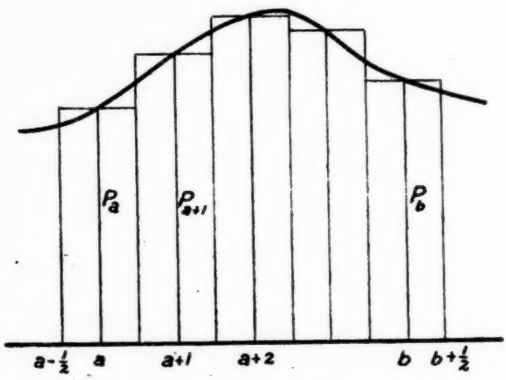
That is, 762 deaths is 38 less than the mean, or $\frac{-38}{\sigma} = -1.3489$ standard units less than the mean.

With $\alpha_3 = 0$ and using the Table of Ordinates of the Pearson Type III Curve, the value of f_t corresponding to $t = -1.35$ is found to be .160383.

$$f_t = .160383$$

$$\therefore \quad P_{762} = \frac{1}{\sigma} f_t = .0354976(.160383) = .005693$$

We shall now consider the following problem: Required the probability that not more than 780 individuals will die within one year, where as before $r = 100,000$, $p = .008$. This means that we must obtain the sum of the 781 terms. $P_0 + P_1 + P_2 + \ldots + P_{780}$



Suppose we represent the sum of the probabilities $P_a + P_{a+1} + P_{a+2} + \ldots + P_b$ by a series of ordinates erected at unit intervals along the x axis, and then construct a series of rectangles having these ordinates as altitudes which bisect the bases of the respective unit bases. Then the area of the first rectangle is $P_a = 1 = P_a$, etc. Thus the sum of the series $P_a + P_{a+1} + \ldots + P_b$ is equal to the total area of all the rectangles and is therefore approximately equal to the area under the frequency curve from $a - \frac{1}{2}$ to $b + \frac{1}{2}$. Therefore the sum of all the probabilities, $P_0 + P_1 + \ldots + P_{780}$ can be computed readily by calculating by means of the Tables of Areas of the Standard Curves, the per cent of the area of the standard curve lying below $x = 780.5$, that is below $\bar{x} = -19.5$, or $t = \frac{-19.5}{\sigma} = -.6922$. For $\alpha_3 = 0$, the per cent of the area of the frequency curve lying below $t = -.69$ is 24.5097. Since the sum of all probabilities from P_0 to $P_{100,000}$ inclusive is 1, and $P_0 + P_1 + \ldots + P_{780}$ represents approximately 24.5097 per cent of the total area under the frequency curve, therefore we estimate that $P_0 + P_1 + P_2 + \ldots + P_{780} = .245097$.

By Table XXIII the correct value is .2454, or the error of our approximation is .0003. Using the values $\alpha_3 = 0$, the per cent of the area lying below $t = -.70$ is found to be 24.1964, using straight line interpolation the per cent below $t = -.6922$ is found to be 24.4408. In the same manner, only using $\alpha_3 = .1$, the per cent of the curve lying below $t = -.6922$ is found to be 24.7105. By using straight line interpolation again for the value of α_3 , it is found that the per cent of the distribution lying below $t = -.6922$, skewness $= .035$, is 24.5352. The error of our approximation is now zero. In general, however, a sufficient degree of accuracy may be obtained without in-

terpolating for either the value of t or α_3.

Next let it be required to find the probability that less than 840 but more than 780 will die within the year, that is, required the value of $P_{781} + P_{782} + \ldots P_{839}$.

We require therefore the per cent of the area of a standard frequency curve lying between $x = 780.5$ and $x = 839.5$, that is between $\bar{x} = -19.5$ and $\bar{x} = 39.5$ or $t = -.69$ to $t = 1.40$.

As has just been shown, 24.5097 per cent of the area of the curve lies below $t = -.69$. Likewise for $\alpha_3 = 0$ the per cent lying below $t = 1.40$ is 91.9243. Consequently 91.9243% − 24.5097%, or 67.4146%, of the area lies between $t = -.69$ and $t = 1.40$. Therefore the probability that less than 840 but more than 780 will die within the year is .674146.

By Table XXIII, the correct value is .9188 − .2454 = .6734.

Summary of Section VI.

If p represent the probability that an event will happen in a single trial, then the probability that the event will happen either 0, 1, 2, ... times during r trials are given by the respective terms of the expansionn of $(q + p)^r$. The distribution of these probabilities or the corresponding expected frequencies is adequately described by the three fundamental functions as follows:

$$M = rp$$

$$\sigma = \sqrt{rp(1-p)}$$

$$\alpha_3 = \frac{1-2p}{\sigma}$$

The probabilities or expected frequencies may be regarded as a distribution that can be reproduced at will by utilizing the Tables of Pearson's Type III Curves, with the fundamental functions computed from the above formulae. In this way the values of isolated probabilities or the sum of any number of consecutive probabilities may be obtained.

EDITORIAL

FUNDAMENTALS OF THE THEORY OF SAMPLING

III. Distribution of Sample m th Moments about the Origin of the Parent Population

As in section I, we shall be concerned with the $\binom{s}{r}$ possible samples, each consisting of r variates, that can be selected from the parent population of s variates $x_1, x_2, \ldots x_r, \ldots x_s$. The m th moment of each sample, computed in each case about the origin of the parent population, may be written

$$\begin{cases} z_1 = \frac{1}{r}\left\{x_1^m + x_2^m + x_3^m + \cdots + x_r^m\right\} \\ z_2 = \frac{1}{r}\left\{x_2^m + x_3^m + x_4^m + \cdots + x_{r+1}^m\right\} \\ \cdots\cdots\cdots\cdots\cdots \\ z_{\binom{s}{r}} = \frac{1}{r}\left\{x_{s-r+1}^m + x_{s-r+2}^m + x_{s-r+3}^m + \cdots + x_s^m\right\} \end{cases}$$

If we write $\frac{x_i^m}{r} = y_i$, it will be observed that the above distribution may be written

$$\begin{cases} z_1 = y_1 + y_2 + y_3 + \cdots + y_r \\ z_2 = y_2 + y_3 + y_4 + \cdots + y_{r+1} \\ \cdots\cdots\cdots\cdots\cdots \\ z_{\binom{s}{r}} = y_{s-r+1} + y_{s-r+2} + y_{s-r+3} + \cdots + y_s \end{cases}$$

and therefore may be regarded as a distribution of the algebraic sums of the respective samples withdrawn from the parent population $y_1, y_2, \ldots y_s$, i. e. $\frac{x_1^m}{r}, \frac{x_2^m}{r}, \ldots \frac{x_s^m}{r}$. Consequently, since

$$\mu'_{n:y} = \frac{\Sigma y_i^n}{N} = \frac{1}{N}\Sigma\frac{x_i^{mn}}{r^n} = \frac{1}{r^n}\mu'_{mn:x}$$

it follows from formulae 1, 2, . . . of section I that

$$(1)\ M_x = r\,M_y = \mu'_{m:x}$$

$$(2)\ \mu_{2:x} = s\{\rho_1-\rho_2\}\mu_{2:y} = s\{\rho_1-\rho_2\}\{\mu'_{2:y}-M_y^2\}$$

$$= s\{\rho_1-\rho_2\}\left\{\frac{\mu'_{2m:x}}{r^2}-\left(\frac{\mu'_{m:x}}{r}\right)^2\right\}$$

$$= \frac{s}{r^2}\{\rho_1-\rho_2\}\{\mu'_{2m:x}-(\mu'_{m:x})^2\}$$

$$(3)\ \mu_{3:x} = s\{\rho_1-3\rho_2+2\rho_3\}\mu_{3:y} = s\{\rho_1-3\rho_2+2\rho_3\}\{\mu'_{3:y}-3M_y\mu'_{2:y}+2M_y^3\}$$

$$= \frac{s}{r^3}\{\rho_1-3\rho_2+2\rho_3\}\{\mu'_{3m:x}-3\mu'_{2m:x}\mu'_{m:x}+2(\mu'_{m:x})^3\}$$

$$(4)\ \mu_{4:x} = \frac{s}{r^4}\{\mu'_{4m:x}-4\mu'_{3m:x}\mu'_{m:x}+6\mu'_{2m:x}(\mu'_{m:x})^2-3(\mu'_{m:x})^4\}$$

$$\{\rho_1-7\rho_2+12\rho_3-6\rho_4\}+3\frac{s^2}{r^4}\{\mu'_{2m:x}-(\mu'_{m:x})^2\}^2\{\rho_2-2\rho_3+\rho_4\}$$

etc.

For the case of sampling from an unlimited supply, we have, permitting s to approach infinity, that corresponding to formulae (18) of section I

$$
(5)\begin{cases}
M_z = \mu'_{m:x} \\
\mu_{2:z} = \frac{1}{r}\left\{ \mu'_{2m:x} - (\mu'_{m:x})^2 \right\} \\
\mu_{3:z} = \frac{1}{r^2}\left\{ \mu'_{3m:x} - 3\mu'_{2m:x}\mu'_{m:x} + 2(\mu'_{m:x})^3 \right\} \\
\mu_{4:z} = \frac{1}{r^3}\left\{ \mu'_{4m:x} - 4\mu'_{3m:x}\mu'_{m:x} + 6\mu'_{m:x}(\mu'_{m:x})^2 - 3(\mu'_{m:x})^4 \right\} \\
\qquad + \frac{3r^{(2)}}{r^4}\left\{ \mu'_{2m:x} - (\mu'_{m:x})^2 \right\}
\end{cases}
$$

etc.

The distribution of sample means may be obtained by placing $m = 1$, yielding

$$
(6)\begin{cases}
M_z = M_x \\
\mu_{2:z} = \frac{1}{r}\mu_{2:x} \\
\mu_{3:z} = \frac{1}{r^2}\mu_{3:x} \\
\mu_{4:z} = \frac{1}{r^3}\mu_{4:x} + \frac{3(r-1)}{r^3}\mu^2_{2:x}
\end{cases}
$$

etc.

These results may be written corresponding to formulae (19) of section I,

$$
(7)\begin{cases}
\mu_{2:z} = \frac{1}{r}\mu_{2:x} \\
\mu_{3:z} = \frac{1}{r^2}\mu_{3:x} \\
\mu_{4:z} - 3\mu^2_{2:z} = \frac{1}{r^3}\left\{ \mu_{4:x} - 3\mu^2_{2:x} \right\} \\
\mu_{5:z} - 10\mu_{3:z}\mu_{2:z} = \frac{1}{r^4}\left\{ \mu_{5:x} - 10\mu_{3:x}\mu_{2:x} \right\} \\
\mu_{6:z} - 15\mu_{4:z}\mu_{2:z} - 10\mu^2_{3:z} + 30\mu^3_{2:z} = \frac{1}{r^5}\left\{ \mu_{6:x} - 15\mu_{4:x}\mu_{2:x} - 10\mu^2_{3:x} + 30\mu^3_{2:x} \right\}
\end{cases}
$$

etc.

The distribution of sample means withdrawn from an infinite parent population is therefore characterized by means of the semi-invariant relation

(8) $$\lambda_{n:x} = \frac{\lambda_{n:x}}{r^{n-1}}$$

and the standard semi-invariants by the relation

(8-a) $$\gamma_{n:x} = \frac{\gamma_{n:x}}{r^{n/2-1}}$$

An interesting result is obtained by considering the special case of formulae (8) for which $n = 2$, and assuming that the parent population is normal. Since for a normal distribution

$$\mu_{2n+1} = 0$$
$$\mu_{2n} = 1 \cdot 3 \cdot 5 \ldots (2n-1)\sigma^{2n}$$

and for any distribution

(9) $$\begin{cases} \mu_2' = \mu_2 + M^2 \\ \mu_3' = \mu_3 + 3M\mu_2 + M^3 \\ \mu_4' = \mu_4 + 4M\mu_3 + 6M^2\mu_2 + M^4 \end{cases}$$

etc.

it follows that for a normal distribution

(10) $$\begin{cases} \mu_2' = \sigma^2 + M^2 \\ \mu_3' = 3M\sigma^2 + M^3 \\ \mu_4' = 3\sigma^4 + 6\sigma^2 M^2 + M^4 \\ \mu_5' = 15M\sigma^4 + 10M^3\sigma^2 + M^5 \end{cases}$$

etc.

and therefore for the distribution of sample second moments about a fixed point in the case of withdrawals from an unlimited "normal" supply, we have, from (5)

1 See formulae 23 and 24, page 117, Vol. I, No. 1, of ANNALS.

$$
(11)\left\{
\begin{aligned}
M_z &= \mu'_{2:x} = \sigma_x^2 + M_x^2 \\
\mu_{2:z} &= \frac{1}{r}\left\{ \mu'_{4:x} - (\mu'_{2:x})^2 \right\} \\
&= \frac{2\sigma^2}{r}\left\{ \sigma_x^2 + 2M_x^2 \right\} \\
\mu_{3:z} &= \frac{1}{r^2}\left\{ \mu_{6:x} - 3\mu'_{4:x}\mu'_{2:x} + 2(\mu'_{2:x})^3 \right\} \\
&= \frac{8\sigma_x^4}{r^2}\left\{ \sigma_x^2 + 3M_x^2 \right\} \\
\mu_{4:z} &= \frac{48\sigma_x^6}{r^3}\left\{ \sigma_x^2 + 4M_x^2 \right\} + \frac{12\sigma_x^4}{r^2}\left\{ \sigma_x^2 + 2M_x^2 \right\}^2 \\
\mu_{5:z} &= \frac{384\sigma_x^8}{r^4}\left\{ \sigma_x^2 + 5M_x^2 \right\} + \frac{160\sigma_x^6}{r^3}\left\{ \sigma_x^2 + 2M_x^2 \right\}\left\{ \sigma_x^2 + 3M_x^2 \right\} \\
\mu_{6:z} &= \frac{3840\sigma_x^{10}}{r^5}\left\{ \sigma_x^2 + 6M_x^2 \right\} + \frac{160\sigma_x^8}{r^4}\left\{ 13\sigma_x^4 + 78\sigma_x^2 M_x^2 + 108M_x^4 \right\} \\
&\quad + \frac{120\sigma_x^6}{r^3}\left\{ \sigma_x^2 + 2M_x^2 \right\}^3
\end{aligned}
\right.
$$

In terms of semi invariants[1]

$$
(12)\left\{
\begin{aligned}
M_z &= \sigma_x^2 + M_x^2 \\
\lambda_{2:z} &= \frac{2\sigma_x^2}{r}(\sigma_x^2 + 2M_x^2) \\
\lambda_{3:z} &= \frac{2^2 2!\,\sigma_x^4}{r^2}(\sigma_x^2 + 3M_x^2) \\
\lambda_{4:z} &= \frac{2^3 3!\,\sigma_x^6}{r^3}(\sigma_x^2 + 4M_x^2) \\
\lambda_{5:z} &= \frac{2^4 4!\,\sigma_x^8}{r^4}(\sigma_x^2 + 5M_x^2) \\
\lambda_{6:z} &= \frac{2^5 5!\,\sigma_x^{10}}{r^5}(\sigma_x^2 + 6M_x^2)
\end{aligned}
\right.
$$

1 Formulae (21), Section I. Page 116, Vol. I, No. 1, of ANNALS.

Apparently the general expression is

$$(13) \qquad \lambda_{n:x} = \frac{2^{n-1}(n-1)!\,\sigma_x^{2n}}{r^{n-1}} \left\{ 1 + n\left(\frac{M_x}{\sigma_x}\right)^2 \right\}$$

If the parent population be normal, and if furthermore $M_x = 0$, then

$$(14) \qquad \lambda_{n:x} = \frac{2^{n-1}(n-1)!\,\sigma_x^{2n}}{r^{n-1}}$$

and the standardized semi-invariants would likewise be

$$(15) \qquad \gamma_{n:x} = \frac{\lambda_{n:x}}{(\lambda_{2:x})^{n/2}} = \left(\frac{2}{r}\right)^{\frac{n}{2}-1} \cdot (n-1)!$$

Again, since

$$\gamma_{3:x} = \alpha_{3:x} = \frac{2^{3/2}}{r^{1/2}}$$

formula (15) may be written

$$(16) \qquad \gamma_{n:x} = \left(\frac{\alpha_3}{2}\right)^{n-2} \cdot (n-1)!$$

On page 196 of Vol. I, No. 2 of the ANNALS it was shown that the standard moments for Pearson's Type III function

$$y = y_0 \left(1 + \frac{\alpha_3}{2} t\right)^{\frac{4}{\alpha_3^2}-1} e^{-\frac{2}{\alpha_3} t}$$

are defined by the recurring relation

$$\alpha_{n+1} = n\left(\alpha_{n-1} + \frac{\alpha_3 \alpha_n}{2}\right),$$

so that $$\alpha_4 = 3\left(1 + \frac{\alpha_3^2}{2}\right)$$

$$\alpha_5 = 2\alpha_3\left(5 + 3\,\frac{\alpha_3^2}{2}\right)$$

$$\alpha_6 = 5\left(3 + 13\,\frac{\alpha_3^2}{2} + 6\,\frac{\alpha_3^4}{4}\right)$$

etc.

The standard semi-invariants of Type III are

$$\gamma_4 = \left(\frac{\alpha_3}{2}\right)^2 \cdot 3!$$

$$\gamma_5 = \left(\frac{\alpha_3}{2}\right)^3 \cdot 4!$$

$$\gamma_6 = \left(\frac{\alpha_3}{2}\right)^4 \cdot 5!$$

etc.

Comparing these results with formula (16) it appears, therefore, that if the parent population be normal *and its mean zero*, the distribution of sample second moments computed about the fixed mean of the parent population will be Pearson't Type III, for r finite. As r approaches infinity, the Type III distribution will approach the Normal Curve as a limit.

To illustrate: If from an infinite population of spherical balls whose diameters formed a normal distribution characterized by M_x and σ_x, samples of r balls each were withdrawn, then if the average area be determined for the balls in each sample, the distribution of these areas, from formula (13), would be described by the relation

$$\lambda_{n:a} = \frac{2^{n-1}(n-1)!\,\sigma_x^{2n}}{r^{n-1}}\left\{1 + n\left(\frac{M_x}{\sigma_x}\right)^2\right\}$$

and if one could conceive of negative diameters of the balls so that $M_x = 0$, then the distribution of areas would be Type III.

If one were to succeed in finding the function whose n th semi-invariant agrees with the above expression, then the law of distribu-

tion for the sample areas would be available. Again by likewise investigating the cases of formulae (5) where $m = 3, 4, 5$, etc., other semi-invariant relations can be found, and these in turn may lead to the discovery of new and important frequency functions. At all events, such sample moments and semi-invariants will generally permit one to express as an infinite series, such as the Gram-Charlier series, the unknown law of distribution.

SECTION IV

The problem of the distribution of sample moments about the origin of the parent population[1] is unfortunately often confused with the problem of the distribution of sample moments computed about the means of the respective samples. The latter problem is more briefly termed sampling *about the mean*. If M_1 and M_2 designate the means of the first two samples respectively, and z_1 and z_2 the second moments of these two samples computed about M_1 and M_2 respectively, then for $m = 2$

$$z_1 = \frac{\sum^{r:1} (x - M_1)^2}{r}$$

$$z_2 = \frac{\sum^{r:2} (x - M_2)^2}{r}$$

where, as before, $\sum^{r:i}$ indicates that the summation extends over the r variates occurring in the i th sample.

In order to sum all values of z_i and z_i^n it is necessary to obtain first another expression for the second moment about the mean, which, although of value in algebraic manipulations, is practically of no value in arithmetic computation. Thus,

$$z_i = \frac{\sum^{r:i} (x - M_i)^2}{r}$$

$$= \frac{\sum^{r:i} x^2 - 2M_i \sum^{r:i} x + rM_i^2}{r}$$

1 Also referred to as the distribution of sample moments *about a fixed point.*

$$= \frac{\sum^{r:i} x^2 - \frac{\left(\sum^{r:i} x\right)^2}{r}}{r}$$

$$= \frac{r\sum^{r:i} x^2 - \left[\sum^{r:i} x^2 + 2\sum^{r:i} x_i x_j\right]}{r^2}$$

(17) $$z_i = \frac{1}{r^2}\left[(r-1)\sum^{r:i} x^2 - 2\sum^{r:i} x_i x_j\right],$$

where $\sum^{r:i} x_i x_j$ designnates the sum of all the terms formed by taking the products of all the variates in the i th sample two at a time.

Then $$M_z = \frac{\sum z_i}{\binom{s}{r}} = \frac{1}{r^2}\left[(r-1)\frac{r}{s}\sum^{s} x^2 - 2\,\frac{r^{(2)}}{s^{(2)}}\sum^{s} x_i x_j\right]$$

by employing the method employed in section I. The above reduces easily as follows:

(18)
$$M_z = \frac{1}{r^2}\left[(r-1)\frac{r}{s}\sum^{s} x^2 - \frac{r^{(2)}}{s^{(2)}}\left\{\left(\sum^{s} x\right)^2 - \sum x^2\right\}\right]$$

$$= \frac{s(r-1)}{r(s-1)}\cdot\left\{\frac{\sum^{s} x^2}{s} - \left(\frac{\sum^{s} x}{s}\right)^2\right\}$$

$$= \frac{s(r-1)}{r(s-1)}\ \mu_{2:x}$$

Whereas the expected value of a sample mean is equal to the mean of the parent population and the expected value of a sample n th moment about a fixed point is equal to the n th moment of the parent population[1], it appears that the expected value of a sample second moment is less than the second moment of the parent population.

A slight digression at this point is desirable. In formula (6) of Section III we found that for the distribution of sample means withdrawn from an infinite parent population,

$$\mu_{2:\bar{x}} = \frac{1}{r}\mu_{2:x}\ .$$

That is, the *standard error of the mean*

1 Formula (1), Section III.

(19) $$\sigma_m = \sigma_z = \frac{\sigma_x}{\sqrt{r}} ,$$

where σ_x denotes the standard deviation of the infinite parent population. By formula (18) above it appears that the expected value of the second sample moment is for $s = \infty$

$$M_2 = \frac{r-1}{r}\mu_{2:x} = \frac{r-1}{r}\sigma_x^2$$

Designating the square root of the expected sample moment by σ'_x, we have that

$$\sigma'_x = \sigma_x\sqrt{\frac{r-1}{r}} \quad , \text{ or } \sigma_x = \sigma'_x\sqrt{\frac{r}{r-1}}$$

and therefore formula (19) may be written

(20) $$\sigma_m = \frac{\sigma'_x}{\sqrt{r-1}}$$

Since the probable error is defined as .6745 σ, we have that the *probable error of the mean*

(21) $$P.E._m = .6745\frac{\sigma_x}{\sqrt{r}} = .6745\frac{\sigma'_x}{\sqrt{r-1}}$$

It should be observed that the expressions for both the standard and probable errors of the mean are expected values when σ' is employed. If one obtains but a single sample and computes its mean and standard deviation, he still has no accurate knowledge regarding the true value of the standard deviation of the parent population. Consequently even the expression

$$P.E._M = .6745\frac{\sigma'_x}{\sqrt{r-1}}$$

is merely an approximation. So far as I know, the true value of the

probable error of the mean has never been found—even upon the assumption that the parent population is normal. Since we have shown that for $s \rightarrow \infty$ the skewness of the samples is only $\frac{1}{\sqrt{r}}$ times the skewness of the parent population, the fact that the parent population is not normal is of no importance compared to the fact that where only functions of the single sample are available, *these* must be substituted as the expected values of the corresponding functions of the unknown parent population

Returning to our problem of describing further the distribution of sample second moments about the mean:

Corresponding to formula (17), one can show by employing symmetric functions that

$$z_i^2 = \frac{1}{r^4}\Big\{(r-1)^2 \sum^{r:i} x^4 - 4(r-1)\sum^{r:i} x_i^3 x_j + 2(r^2 - 2r+3)\sum^{r:i} x_i^2 x_j^2 \tag{22}$$

$$- 4(r-3)\sum^{r:i} x_i^2 x_j x_k + 24\sum^{r:i} x_i x_j x_k x_l\Big\}$$

and therefore

$$\mu_{2:z} = \frac{\sum z_i^2}{\binom{s}{r}} - M_z^2$$

$$(23) \qquad = \frac{s(r-1)(s-r)}{r^3(s-1)(s-2)(s-3)} \cdot \Big\{(s-1)(rs-s-r-1)\mu_{4:x}$$

$$+ \left[(3-r)^2 s - 6s + 3r + 3\right]\mu_{2:x}^2\Big\}$$

For $s=\infty$ this becomes

$$\mu_{2:z} = \frac{r-1}{r^3}\left[(r-1)\mu_{4:x} - (r-3)\mu_{2:x}^2\right]$$

$$(24) \qquad = \frac{(r-1)\sigma^4}{r^3}\left[(r-1)\alpha_{4:x} - (r-3)\right]$$

In a thesis, C. H. Richardson[1] has shown that when $s=\infty$

1 Submitted in 1927 to the University of Michigan. The balance of this section is a synopsis of one part of this thesis.

$$(25)\quad \mu_{3:x} = \frac{(r-1)\sigma^6}{r^5}\Big[(r-1)^2\alpha_{6:x} - 3(r-1)(r-5)\alpha_{4:x}$$

$$-2(3r^2-6r+5)\alpha_3^2 + 2(r^2-12r+15)\Big]$$

$$(26)\quad \mu_{4:x} = \frac{(r-1)\sigma^8}{r^7}\Big[(r-1)^3\alpha_{8:x} - 8(r-1)(3r^2-6r+7)\alpha_{5:x}\alpha_{3:x}$$

$$+(3r^4-12r^3+42r^2-60r+35)\alpha_{4:x}^2 - 4(r-1)^2(r-7)\alpha_{6:x}$$

$$-6(r^4-7r^3+49r^2-105r+70)\alpha_{4:x}$$

$$+16(6r^3-27r^2+50r-35)\alpha_{3:x}^2$$

$$+3(r^4-9r^3+93r^2-255r+210)\Big]$$

$$(27)\quad \mu_{5:x} = \frac{(r-1)\sigma^{10}}{r^9}\Big[(r-1)^4\alpha_{10:x} - 5(r-1)^3(r-9)\alpha_{8:x}$$

$$-40(r-1)^2(r^2-2r+3)\alpha_{7:x}\alpha_{3:x}$$

$$+10(r-1)(r^4-4r^3+18r^2-28r+21)\alpha_{6:x}\alpha_{4:x}$$

$$-10(3r^5-27r^4+162r^3-450r^2+595r-315)\alpha_{4:x}^2$$

$$-20(r-2)(3r^4-24r^3+80r^2-140r+105)\alpha_{4:x}\alpha_{3:x}^2$$

$$+10(5r^5-64r^4+572r^3-2070r^2+3255r-1890)\alpha_{3:x}^2$$

$$-4(5r^5-86r^4+1050r^3-4620r^2+8505r-5670)$$

$$-10(r-1)(r^4-7r^3+65r^2-161r+126)\alpha_{6:x}$$

$$-2(15r^4-60r^3+130r^2-140r+63)\alpha_{5:x}^2$$

$$-80(6r^4-36r^3+97r^2-126r+63)\alpha_{5:x}\alpha_{3:x}\Big]$$

$$
\begin{aligned}
(28)\ \mu_{6:x} = \frac{(r-1)\,\sigma^{12}}{r^{11}} \Big[& (r-1)^5\,\alpha_{12:x} - 6(r^5-15r^4+50r^3-70r^2+45r-11)\,\alpha_{10:x} \\
& - 20(3r^5-15r^4+38r^3-54r^2+39r-11)\,\alpha_{9:x}\,\alpha_{3:x} \\
& + 15(r^6-6r^5+31r^4-84r^3+127r^2-102r+33)\,\alpha_{8:x}\,\alpha_{4:x} \\
& - 15(r^6-9r^5+96r^4-394r^3+729r^2-621r+198)\,\alpha_{8:x} \\
& + 2(5r^6-30r^5+165r^4-460r^3+735r^2-630r+231)\,\alpha^2_{6:x} \\
& - 120(r^6-11r^5+81r^4-294r^3+567r^2-567r+231)\,\alpha_{6:x}\,\alpha_{4:x} \\
& + 15(r^7-8r^6+51r^5-258r^4+815r^3-1540r^2+1645r-770)\,\alpha^3_{4:x} \\
& + 20(5r^6-75r^5+828r^4-3938r^3+9009r^2-9891r+4158)\,\alpha_{6:x} \\
& + 5(9r^7-159r^6+2436r^5-26130r^4+135885r^3-35941r^2 \\
& \qquad + 474390r-24980)\,\alpha_{4:x} \\
& - 5(9r^7-126r^6+1413r^5-11214r^4+47355r^3-107730r^2 \\
& \qquad + 127575r-62370)\,\alpha^2_{4:x} \\
& - 24(5r^5-25r^4+70r^3-110r^2+93r-33)\,\alpha_{7:x}\,\alpha_{5:x} \\
& + 480(2r^5-15r^4+52r^3-96r^2+90r-33)\,\alpha_{7:x}\,\alpha_{3:x} \\
& - 40(3r^6-39r^5+206r^4-616r^3+1113r^2-1113r+462)\,\alpha_{6:x}\,\alpha^2_{3:x} \\
& + 24(30r^5-225r^4+820r^3-1610r^2+1638r-693)\,\alpha^2_{5:x} \\
& - 120(3r^6-33r^5+172r^4-530r^3+987r^2-1029r+462)\,\alpha_{5:x}\,\alpha_{4:x}\,\alpha_{3:x} \\
& + 40(51r^6-435r^5+1896r^4-5218r^3+9191r^2-9387r+4158)\,\alpha_{5:x}\,\alpha_{3:x} \\
& + 600(3r^6-45r^5+294r^4-1076r^3+2317r^2-2751r+1386)\,\alpha_{4:x}\,\alpha^2_{3:x} \\
& - 80(21r^6-558r^5+5012r^4-22820r^3+57445r^2-76230r \\
& \qquad + 41580)\,\alpha^2_{3:x} \\
& + 40(9r^6-105r^5+564r^4-1830r^3+3745r^2-4445r+2310)\,\alpha^4_{3:x} \\
& - 5(3r^7-59r^6+1136r^5-15642r^4+96135r^3-290115r^2 \\
& \qquad + 429030r-249480)\Big]
\end{aligned}
$$

If the parent population be normal, that is if

$$\alpha_{2n+1} = 0$$

$$\alpha_{2n} = \frac{(2n)!}{2^n \cdot n!}$$

the preceding formulae yield on reduction

$$(29) \qquad \mu_{2:z} = \frac{2(r-1)}{r^2}\,\mu^2_{2:x}$$

$$(30) \qquad \mu_{3:z} = \frac{8(r-1)}{r^3}\,\mu^3_{2:x}$$

$$(31) \qquad \mu_{4:z} = \frac{12(r-1)(r+3)}{r^4}\,\mu^4_{2:x}$$

$$(32) \qquad \mu_{5:z} = \frac{32(r-1)(5r+7)}{r^5}\,\mu^5_{2:x}$$

$$(33) \qquad \mu_{6:z} = \frac{40(r-1)(3r^2+46r+47)}{r^6}\,\mu^6_{2:x}$$

These may be written in turn

$$(34) \qquad \begin{cases} \alpha_{3:z} = \dfrac{4}{\sqrt{2(r-1)}} \\[2ex] \alpha_{4:z} = \dfrac{3(r+3)}{r-1} \\[2ex] \alpha_{5:z} = \dfrac{8(5r+7)}{\sqrt{2}\,(r-1)^{3/2}} \\[2ex] \alpha_{6:z} = \dfrac{5(3r^2+46r+47)}{(r-1)^2} \end{cases}$$

For the corresponding standard semi-invariants

$$\gamma_3 = \alpha_3 = \frac{4}{\sqrt{2(r-1)}}$$

$$\gamma_4 = \alpha_4 - 3 = \frac{12}{r-1} = \left(\frac{\alpha_3}{2}\right)^2 \cdot 3!$$

$$\gamma_5 = \alpha_5 - 10\alpha_3 = \frac{96}{\sqrt{2}\,(r-1)^{3/2}} = \left(\frac{\alpha_3}{2}\right)^3 \cdot 4!$$

$$\gamma_6 = \alpha_6 - 15\alpha_4 - 10\alpha_3^2 + 30 = \frac{480}{(r-1)^2} = \left(\frac{\alpha_3}{2}\right)^4 \cdot 5!$$

These results show that so far as the sixth standard semi-invariant the distribution of sample second moments about the mean is Type III, irrespective of the mean of the parent population.

It is to be regretted that many of the results presented here have never been generalized for moments of any order. The methods presented have been chosen for two reasons: first, they permit one with no knowledge of calculus to achieve somewhat of an understanding into the theory of sampling; and secondly, they yield results of sampling from a finite parent population--a problem of considerable practical importance.

The results of sampling from an infinite population may be obtained more readily and with far greater elegance and rigor by employing the method of semi-invariants.

H. C. C.

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TABLE III

DERIVATIVES OF THE STANDARDIZED TYPE III FUNCTION

$$y = y_0 \left(1 + \frac{\alpha_3}{2} t\right)^{\frac{4}{\alpha_3^2} - 1} e^{-\frac{2}{\alpha_3} t}$$

t	SKEWNESS .0	.1	.2	.3	.4	.5	t
-9.90							-9.90
-9.80							-9.80
-9.70							-9.70
-9.60							-9.60
-9.50							-9.50
-9.40							-9.40
-9.30							-9.30
-9.20							-9.20
-9.10							-9.10
-9.00							-9.00
-8.90							-8.90
-8.80							-8.80
-8.70							-8.70
-8.60							-8.60
-8.50							-8.50
-8.40							-8.40
-8.30							-8.30
-8.20							-8.20
-8.10							-8.10
-8.00							-8.00
-7.90							-7.90
-7.80							-7.80
-7.70							-7.70
-7.60							-7.60
-7.50							-7.50
-7.40							-7.40
-7.30							-7.30
-7.20							-7.20
-7.10							-7.10
-7.00							-7.00
-6.90							-6.90
-6.80							-6.80
-6.70							-6.70
-6.60							-6.60
-6.50							-6.50
-6.40							-6.40
-6.30							-6.30
-6.20							-6.20
-6.10							-6.10
6.00							-6.00
-5.90							-5.90
-5.80							-5.80
-5.70							-5.70
-5.60							-5.60
-5.50	.000001						-5.50
-5.40	.000001						-5.40
-5.30	.000002						-5.30
-5.20	.000003						-5.20
-5.10	.000005	.000001					5.10
-5.00	.000007	.000001					-5.00

	SKEWNESS						
t	.6	.7	.8	.9	1.0	1.1	t
–9.90							–9.90
–9.80							–9.80
–9.70							–9.70
–9.60							–9.60
–9.50							–9.50
–9.40							–9.40
–9.30							–9.30
–9.20							–9.20
–9.10							–9.10
–9.00							–9.00
–8.90							–8.90
–8.80							–8.80
–8.70							–8.70
–8.60							–8.60
–8.50							–8.50
–8.40							–8.40
–8.30							–8.30
–8.20							–8.20
–8.10							–8.10
–8.00							–8.00
–7.90							–7.90
–7.80							–7.80
–7.70							–7.70
–7.60							–7.60
–7.50							–7.50
–7.40							–7.40
–7.30							–7.30
–7.20							–7.20
–7.10							–7.10
–7.00							–7.00
–6.90							–6.90
–6.80							–6.80
–6.70							–6.70
–6.60							–6.60
–6.50							–6.50
–6.40							–6.40
–6.30							–6.30
–6.20							–6.20
–6.10							–6.10
–6.00							–6.00
–5.90							–5.90
–5.80							–5.80
–5.70							–5.70
–5.60							–5.60
–5.50							–5.50
–5.40							–5.40
–5.30							–5.30
–5.20							–5.20
–5.10							–5.10
–5.00							–5.00

t	SKEWNESS .0	.1	.2	.3	.4	.5	t
-4.90	.000012	.000002					-4.90
-4.80	.000019	.000003					-4.80
-4.70	.000030	.000006					-4.70
-4.60	.000047	.000011	.000001				-4.60
-4.50	.000072	.000019	.000002				-4.50
-4.40	.000110	.000032	.000005				-4.40
-4.30	.000166	.000055	.000009				-4.30
-4.20	.000248	.000090	.000019	.000001			-4.20
-4.10	.000366	.000147	.000036	.000003			-4.10
-4.00	.000535	.000235	.000066	.000008			-4.00
-3.90	.000775	.000370	.000120	.000018			-3.90
-3.80	.001109	.000573	.000212	.000040	.000001		-3.80
-3.70	.001572	.000874	.000364	.000087	.000005		-3.70
-3.60	.002203	.001313	.000609	.000177	.000017		-3.60
-3.50	.003054	.001941	.000995	.000344	.000049		-3.50
-3.40	.004190	.002825	.001586	.000641	.000128	.000003	-3.40
-3.30	.005684	.004049	.002470	.001146	.000302	.000017	-3.30
-3.20	.007629	.005717	.003760	.001974	.000661	.000069	-3.20
-3.10	.010127	.007950	.005598	.003277	.001344	.000234	-3.10
-3.00	.013296	.010890	.008154	.005256	.002561	.000662	-3.00
-2.90	.017262	.014698	.011630	.008159	.004601	.001623	-2.90
-2.80	.022163	.019545	.016246	.012275	.007834	.003540	-2.80
-2.70	.028136	.025608	.022239	.017924	.012700	.006991	-2.70
-2.60	.035316	.033062	.029844	.025438	.019681	.012686	-2.60
-2.50	.043821	.042064	.039276	.035127	.029254	.021390	-2.50
-2.40	.053747	.052738	.050708	.047251	.041831	.033818	-2.40
-2.30	.065152	.065158	.064244	.061970	.057693	.050508	-2.30
-2.20	.078044	.079330	.079895	.079313	.076923	.071689	-2.20
-2.10	.092366	.095172	.097548	.099136	.099346	.097186	-2.10
-2.00	.107982	.112500	.116952	.121097	.124495	.126364	-2.00
-1.90	.124670	.131014	.137704	.144643	.151607	.158125	-1.90
-1.80	.142110	.150291	.159241	.169016	.179627	.190976	-1.80
-1.70	.159883	.169790	.180854	.193275	.207285	.223136	-1.70
-1.60	.177473	.188856	.201707	.216340	.233159	.252688	-1.60
-1.50	.194276	.206743	.220876	.237054	.255782	.277745	-1.50
-1.40	.209618	.222645	.237390	.254249	.273749	.296608	-1.40
-1.30	.222779	.235726	.250291	.266832	.285824	.307909	-1.30
-1.20	.233023	.245172	.258688	.273853	.291033	.310712	-1.20
-1.10	.239637	.250235	.261823	.274582	.288736	.304576	-1.10
-1.00	.241971	.250281	.259120	.268557	.278673	.289564	-1.00
- .90	.239477	.244839	.250233	.255628	.260978	.266223	- .90
- .80	.231753	.233638	.235076	.235963	.236165	.235512	- .80
- .70	.218578	.216634	.213840	.210049	.205082	.198720	- .70
- .60	.199935	.194029	.186988	.178657	.168852	.157358	- .60
- .50	.176033	.166270	.155237	.142796	.128795	.113055	- .50
- .40	.147308	.134036	.119517	.103653	.086338	.067454	- .40
- .30	.114416	.098208	.080927	.062522	.042938	.022119	- .30
- .20	.078209	.059827	.040672	.020733	.000000	-.021537	- .20
- .10	.039695	.020043	.000000	-.020417	-.041192	-.062305	- .10
.00	.000000	-.019943	-.039861	-.059729	-.079523	-.099218	.00

t	SKEWNESS						t
	.6	.7	.8	.9	1.0	1.1	
-4.90							-4.90
-4.80							-4.80
-4.70							-4.70
-4.60							-4.60
-4.50							-4.50
-4.40							-4.40
-4.30							-4.30
-4.20							-4.20
-4.10							-4.10
-4.00							-4.00
-3.90							-3.90
-3.80							-3.80
-3.70							-3.70
-3.60							-3.60
-3.50							-3.50
-3.40							-3.40
-3.30							-3.30
-3.20							-3.20
-3.10	.000001						-3.10
-3.00	.000019						-3.00
-2.90	.000137						-2.90
-2.80	.000628						-2.80
-2.70	.002066	.000036					-2.70
-2.60	.005395	.000536					-2.60
-2.50	.011851	.002918					-2.50
-2.40	.022763	.009573	.000364				-2.40
-2.30	.039266	.023160	.005119				-2.30
-2.20	.062013	.045673	.021161	.000121			-2.20
-2.10	.090955	.077731	.052860	.013697			-2.10
-2.00	.125234	.118306	.100022	.059683			-2.00
-1.90	.163228	.164867	.158494	.133204	.061132		-1.90
-1.80	.202703	.213833	.221819	.219728	.185902	.031381	-1.80
-1.70	.241060	.261146	.282948	.304226	.316115	.277318	-1.70
-1.60	.275619	.302858	.335559	.375014	.421770	.470105	-1.60
-1.50	.303895	.335595	.374866	.424825	.490506	.580401	-1.50
-1.40	.323832	.356881	.397950	.450518	.520464	.618649	-1.40
-1.30	.333974	.365290	.403738	.452231	.515552	.602164	-1.30
-1.20	.333548	.360455	.392746	.432376	.482398	.547863	-1.20
-1.10	.322482	.342963	.366717	.394732	.428455	.470099	-1.10
-1.00	.301350	.314173	.328212	.343691	.360894	.380193	-1.00
- .90	.271274	.276005	.280235	.283702	.286020	.286608	- .90
- .80	.233788	.230710	.225910	.218898	.209014	.195355	- .80
- .70	.190694	.180672	.168238	.152867	.133891	.110444	- .70
- .60	.143917	.128225	.109914	.088542	.063567	.034327	- .60
- .50	.095375	.075522	.053226	.028174	.000000	-.031726	- .50
- .40	.046869	.024439	.000000	-.026629	-.055654	-.087304	- .40
- .30	.000000	-.023484	-.048406	-.074843	-.102879	-.132606	- .30
- .20	-.043888	-.067064	-.091075	-.115932	-.141646	-.168230	- .20
- .10	-.083738	-.105473	-.127489	-.149766	-.172285	-.195025	- .10
.00	-.118789	-.138212	-.157465	-.176524	-.195367	-.213972	.00

t	SKEWNESS .0	.1	.2	.3	.4	.5	t
.00	.000000	–.019943	–.039861	–.059729	–.079523	–.099218	.00
.10	–.039695	–.058941	–.077764	–.096151	–.114088	–.131563	.10
.20	–.078209	–.095827	–.112693	–.128817	–.144210	–.158882	.20
.30	–.114416	–.129600	–.143805	–.157072	–.169444	–.180957	.30
.40	–.147308	–.159421	–.170454	–.180482	–.189571	–.197783	.40
.50	–.176033	–.184644	–.192211	–.198827	–.204577	–.209535	.50
.60	–.199935	–.204840	–.208859	–.212093	–.214627	–.216536	.60
.70	–.218578	–.219795	–.220388	–.220442	–.220032	–.219219	.70
.80	–.231753	–.229507	–.226967	–.224189	–.221217	–.218089	.80
.90	–.239477	–.234167	–.228926	–.223762	–.218684	–.213694	.90
1.00	–.241971	–.234132	–.226714	–.219676	–.212981	–.206595	1.00
1.10	–.239637	–.229889	–.220873	–.212494	–.204674	–.197346	1.10
1.20	–.233023	–.222022	–.211994	–.202799	–.194322	–.186470	1.20
1.30	–.222779	–.211174	–.200694	–.191168	–.182458	–.174450	1.30
1.40	–.209618	–.198009	–.187582	–.178152	–.169571	–.161717	1.40
1.50	–.194276	–.183183	–.173236	–.164256	–.156099	–.148647	1.50
1.60	–.177473	–.167315	–.158186	–.149931	–.142423	–.135559	1.60
1.70	–.159883	–.150967	–.142899	–.135563	–.128862	–.122712	1.70
1.80	–.142110	–.134628	–.127773	–.121475	–.115674	–.110315	1.80
1.90	–.124670	–.118709	–.113132	–.107924	–.103063	–.098523	1.90
2.00	–.107982	–.103534	–.099229	–.095105	–.091177	–.087449	2.00
2.10	–.092366	–.089346	–.086249	–.083156	–.080118	–.077165	2.10
2.20	–.078044	–.076312	–.074312	–.072165	–.069947	–.067712	2.20
2.30	–.065152	–.064527	–.063487	–.062176	–.060692	–.059103	2.30
2.40	–.053747	–.054029	–.053795	–.053200	–.052351	–.051327	2.40
2.50	–.043821	–.044806	–.045221	–.045215	–.044900	–.044359	2.50
2.60	–.035316	–.036811	–.037720	–.038180	–.038300	–.038160	2.60
2.70	–.028136	–.029964	–.031226	–.032039	–.032498	–.032681	2.70
2.80	–.022163	–.024172	–.025661	–.026723	–.027436	–.027870	2.80
2.90	–.017262	–.019326	–.020937	–.022158	–.023050	–.023670	2.90
3.00	–.013296	–.015318	–.016964	–.018268	–.019273	–.020023	3.00
3.10	–.010127	–.012037	–.013651	–.014978	–.016042	–.016874	3.10
3.20	–.007629	–.009379	–.010911	–.012214	–.013293	–.014168	3.20
3.30	–.005684	–.007247	–.008665	–.009908	–.010969	–.011854	3.30
3.40	–.004190	–.005554	–.006837	–.007996	–.009013	–.009884	3.40
3.50	–.003054	–.004222	–.005361	–.006422	–.007376	–.008214	3.50
3.60	–.002203	–.003184	–.004178	–.005132	–.006013	–.006805	3.60
3.70	–.001572	–.002382	–.003236	–.004082	–.004883	–.005620	3.70
3.80	–.001109	–.001768	–.002492	–.003232	–.003951	–.004627	3.80
3.90	–.000775	–.001303	–.001908	–.002547	–.003185	–.003799	3.90
4.00	–.000535	–.000952	–.001452	–.001999	–.002559	–.003110	4.00
4.10	–.000366	–.000691	–.001100	–.001561	–.002049	–.002539	4.10
4.20	–.000248	–.000498	–.000828	–.001215	–.001635	–.002067	4.20
4.30	–.000166	–.000356	–.000620	–.000941	–.001301	–.001679	4.30
4.40	–.000110	–.000253	–.000462	–.000726	–.001031	–.001360	4.40
4.50	–.000072	–.000178	–.000342	–.000558	–.000815	–.001099	4.50
4.60	–.000047	–.000125	–.000252	–.000428	–.000642	–.000886	4.60
4.70	–.000030	–.000087	–.000185	–.000326	–.000505	–.000713	4.70
4.80	–.000019	–.000060	–.000135	–.000248	–.000396	–.000572	4.80
4.90	–.000012	–.000041	–.000098	–.000188	–.000309	–.000458	4.90

t	SKEWNESS .6	.7	.8	.9	1.0	1.1	t
.00	-.118789	-.138212	-.157465	-.176524	-.195367	-.213972	.00
.10	-.148563	-.165078	-.181098	-.196613	-.211618	-.226104	.10
.20	-.172846	-.186112	-.198692	-.210598	-.221843	-.232441	.20
.30	-.191649	-.201555	-.210707	-.219137	-.226877	-.233955	.30
.40	-.205174	-.211799	-.217704	-.222936	-.227536	-.231545	.40
.50	-.213769	-.217340	-.220302	-.222707	-.224598	-.226019	.50
.60	-.217887	-.218739	-.219142	-.219142	-.218780	-.218092	.60
.70	-.218055	-.216587	-.214854	-.212890	-.210726	-.208386	.70
.80	-.214834	-.211479	-.208044	-.204548	-.201006	-.197431	.80
.90	-.208794	-.203987	-.199272	-.194648	-.190114	-.185671	.90
1.00	-.200493	-.194649	-.189043	-.183655	-.178470	-.173473	1.00
1.10	-.190456	-.183956	-.177806	-.171972	-.166424	-.161137	1.10
1.20	-.179166	-.172343	-.165948	-.159935	-.154263	-.148900	1.20
1.30	-.167053	-.160191	-.153798	-.147823	-.142218	-.136946	1.30
1.40	-.154494	-.147820	-.141627	-.135859	-.130469	-.125416	1.40
1.50	-.141806	-.135497	-.129653	-.124221	-.119153	-.114410	1.50
1.60	-.129254	-.123437	-.118049	-.113040	-.108368	-.103997	1.60
1.70	-.117047	-.111807	-.106944	-.102415	-.098185	-.094223	1.70
1.80	-.105348	-.100733	-.096431	-.092411	-.088645	-.085109	1.80
1.90	-.094278	-.090302	-.086573	-.083069	-.079770	-.076660	1.90
2.00	-.083916	-.080572	-.077405	-.074407	-.071565	-.068870	2.00
2.10	-.074314	-.071572	-.068944	-.066428	-.064021	-.061721	2.10
2.20	-.065493	-.063313	-.061186	-.059120	-.057120	-.055189	2.20
2.30	-.057456	-.055786	-.054116	-.052462	-.050835	-.049243	2.30
2.40	-.050187	-.048971	-.047709	-.046425	-.045135	-.043849	2.40
2.50	-.043655	-.042835	-.041933	-.040977	-.039985	-.038973	2.50
2.60	-.037824	-.037342	-.036750	-.036078	-.035348	-.034578	2.60
2.70	-.032648	-.032449	-.032119	-.031691	-.031187	-.030627	2.70
2.80	-.028079	-.028110	-.027999	-.027776	-.027464	-.027084	2.80
2.90	-.024066	-.024280	-.024346	-.024293	-.024143	-.023915	2.90
3.00	-.020558	-.020913	-.021120	-.021204	-.021187	-.021087	3.00
3.10	-.017505	-.017965	-.018279	-.018472	-.018563	-.018568	3.10
3.20	-.014860	-.015393	-.015787	-.016063	-.016238	-.016329	3.20
3.30	-.012578	-.013156	-.013606	-.013944	-.014184	-.014342	3.30
3.40	-.010616	-.011218	-.011703	-.012084	-.012373	-.012582	3.40
3.50	-.008935	-.009543	-.010046	-.010455	-.010778	-.011026	3.50
3.60	-.007501	-.008100	-.008609	-.009032	-.009377	-.009652	3.60
3.70	-.006280	-.006862	-.007364	-.007791	-.008148	-.008440	3.70
3.80	-.005246	-.005800	-.006288	-.006711	-.007071	-.007374	3.80
3.90	-.004371	-.004893	-.005361	-.005773	-.006130	-.006436	3.90
4.00	-.003634	-.004120	-.004563	-.004959	-.005309	-.005612	4.00
4.10	-.003014	-.003463	-.003878	-.004255	-.004592	-.004889	4.10
4.20	-.002495	-.002906	-.003291	-.003646	-.003969	-.004256	4.20
4.30	-.002061	-.002434	-.002789	-.003121	-.003426	-.003702	4.30
4.40	-.001699	-.002035	-.002360	-.002669	-.002955	-.003218	4.40
4.50	-.001397	-.001699	-.001995	-.002279	-.002547	-.002795	4.50
4.60	-.001147	-.001416	-.001684	-.001944	-.002193	-.002425	4.60
4.70	-.000940	-.001179	-.001420	-.001657	-.001886	-.002104	4.70
4.80	-.000769	-.000979	-.001195	-.001411	-.001621	-.001823	4.80
4.90	-.000628	-.000813	-.001005	-.001200	-.001393	-.001579	4.90

t	SKEWNESS .0	.1	.2	.3	.4	.5	t
5.00	-.000007	-.000028	-.000071	-.000142	-.000241	-.000366	5.00
5.10	-.000005	-.000019	-.000051	-.000106	-.000187	-.000292	5.10
5.20	-.000003	-.000013	-.000036	-.000080	-.000145	-.000232	5.20
5.30	-.000002	-.000008	-.000026	-.000060	-.000112	-.000184	5.30
5.40	-.000001	-.000006	-.000018	-.000044	-.000087	-.000146	5.40
5.50	-.000001	-.000004	-.000013	-.000033	-.000067	-.000116	5.50
5.60		-.000002	-.000009	-.000024	-.000051	-.000091	5.60
5.70		-.000002	-.000006	-.000018	-.000039	-.000072	5.70
5.80		-.000001	-.000004	-.000013	-.000030	-.000057	5.80
5.90		-.000001	-.000003	-.000010	-.000023	-.000044	5.90
6.00			-.000002	-.000007	-.000017	-.000035	6.00
6.10			-.000001	-.000005	-.000013	-.000027	6.10
6.20			-.000001	-.000004	-.000010	-.000021	6.20
6.30			-.000001	-.000003	-.000008	-.000017	6.30
6.40				-.000002	-.000006	-.000013	6.40
6.50				-.000001	-.000004	-.000010	6.50
6.60				-.000001	-.000003	-.000008	6.60
6.70				-.000001	-.000002	-.000006	6.70
6.80				-.000001	-.000002	-.000005	6.80
6.90					-.000001	-.000004	6.90
7.00					-.000001	-.000003	7.00
7.10					-.000001	-.000002	7.10
7.20					-.000001	-.000002	7.20
7.30						-.000001	7.30
7.40						-.000001	7.40
7.50						-.000001	7.50
7.60						-.000001	7.60
7.70							7.70
7.80							7.80
7.90							7.90
8.00							8.00
8.10							8.10
8.20							8.20
8.30							8.30
8.40							8.40
8.50							8.50
8.60							8.60
8.70							8.70
8.80							8.80
8.90							8.90
9.00							9.00
9.10							9.10
9.20							9.20
9.30							9.30
9.40							9.40
9.50							9.50
9.60							9.60
9.70							9.70
9.80							9.80
9.90							9.90

t	SKEWNESS .6	.7	.8	.9	1.0	1.1	t
5.00	-.000512	-.000674	-.000845	-.001020	-.001195	-.001367	5.00
5.10	-.000417	-.000557	-.000709	-.000866	-.001025	-.001182	5.10
5.20	-.000339	-.000461	-.000594	-.000735	-.000878	-.001022	5.20
5.30	-.000275	-.000380	-.000497	-.000623	-.000752	-.000883	5.30
5.40	-.000223	-.000314	-.000416	-.000527	-.000644	-.000763	5.40
5.50	-.000180	-.000258	-.000348	-.000446	-.000551	-.000658	5.50
5.60	-.000145	-.000212	-.000290	-.000377	-.000471	-.000568	5.60
5.70	-.000117	-.000174	-.000242	-.000319	-.000402	-.000490	5.70
5.80	-.000094	-.000143	-.000202	-.000269	-.000343	-.000422	5.80
5.90	-.000076	-.000117	-.000168	-.000227	-.000293	-.000364	5.90
6.00	-.000061	-.000096	-.000140	-.000191	-.000250	-.000313	6.00
6.10	-.000049	-.000079	-.000116	-.000161	-.000213	-.000269	6.10
6.20	-.000039	-.000064	-.000097	-.000136	-.000181	-.000232	6.20
6.30	-.000031	-.000052	-.000080	-.000114	-.000154	-.000199	6.30
6.40	-.000025	-.000043	-.000066	-.000096	-.000131	-.000171	6.40
6.50	-.000020	-.000035	-.000055	-.000081	-.000112	-.000147	6.50
6.60	-.000016	-.000028	-.000046	-.000068	-.000095	-.000126	6.60
6.70	-.000013	-.000023	-.000038	-.000057	-.000081	-.000109	6.70
6.80	-.000010	-.000019	-.000031	-.000048	-.000069	-.000093	6.80
6.90	-.000008	-.000015	-.000026	-.000040	-.000058	-.000080	6.90
7.00	-.000006	-.000012	-.000021	-.000034	-.000049	-.000069	7.00
7.10	-.000005	-.000010	-.000018	-.000028	-.000042	-.000059	7.10
7.20	-.000004	-.000008	-.000014	-.000024	-.000035	-.000050	7.20
7.30	-.000003	-.000007	-.000012	-.000020	-.000030	-.000043	7.30
7.40	-.000003	-.000005	-.000010	-.000016	-.000025	-.000037	7.40
7.50	-.000002	-.000004	-.000008	-.000014	-.000022	-.000032	7.50
7.60	-.000002	-.000003	-.000007	-.000012	-.000018	-.000027	7.60
7.70	-.000001	-.000003	-.000005	-.000010	-.000015	-.000023	7.70
7.80	-.000001	-.000002	-.000005	-.000008	-.000013	-.000020	7.80
7.90	-.000001	-.000002	-.000004	-.000007	-.000011	-.000017	7.90
8.00	-.000001	-.000001	-.000003	-.000006	-.000009	-.000014	8.00
8.10		-.000001	-.000002	-.000005	-.000008	-.000012	8.10
8.20		-.000001	-.000002	-.000004	-.000007	-.000011	8.20
8.30		-.000001	-.000002	-.000003	-.000006	-.000009	8.30
8.40		-.000001	-.000001	-.000003	-.000005	-.000008	8.40
8.50			-.000001	-.000002	-.000004	-.000007	8.50
8.60			-.000001	-.000002	-.000003	-.000006	8.60
8.70			-.000001	-.000002	-.000003	-.000005	8.70
8.80			-.000001	-.000001	-.000002	-.000004	8.80
8.90			-.000001	-.000001	-.000002	-.000003	8.90
9.00				-.000001	-.000002	-.000003	9.00
9.10				-.000001	-.000001	-.000003	9.10
9.20				-.000001	-.000001	-.000002	9.20
9.30				-.000001	-.000001	-.000002	9.30
9.40					-.000001	-.000002	9.40
9.50					-.000001	-.000001	9.50
9.60					-.000001	-.000001	9.60
9.70					-.000001	-.000001	9.70
9.80						-.000001	9.80
9.90						-.000001	9.90

t	SKEWNESS						t
	.0	.1	.2	.3	.4	.5	
10.00							10.00
10.10							10.10
10.20							10.20
10.30							10.30
10.40							10.40
10.50							10.50
10.60							10.60
10.70							10.70
10.80							10.80
10.90							10.90
11.00							11.00
11.10							11.10
11.20							11.20
11.30							11.30
11.40							11.40
11.50							11.50
11.60							11.60
11.70							11.70
11.80							11.80
11.90							11.90
12.00							12.00
12.10							12.10
12.20							12.20
12.30							12.30
12.40							12.40
12.50							12.50
12.60							12.60
12.70							12.70
12.80							12.80
12.90							12.90
13.00							13.00
13.10							13.10
13.20							13.20
13.30							13.30
13.40							13.40
13.50							13.50
13.60							13.60
13.70							13.70
13.80							13.80
13.90							13.90
14.00							14.00
14.10							14.10
14.20							14.20
14.30							14.30
14.40							14.40
14.50							14.50
14.60							14.60
14.70							14.70
14.80							14.80
14.90							14.90

t	SKEWNESS .6	.7	.8	.9	1.0	1.1	t
10.00						-.000001	10.00
10.10						-.000001	10.10
10.20							10.20
10.30							10.30
10.40							10.40
10.50							10.50
10.60							10.60
10.70							10.70
10.80							10.80
10.90							10.90
11.00							11.00
11.10							11.10
11.20							11.20
11.30							11.30
11.40							11.40
11.50							11.50
11.60							11.60
11.70							11.70
11.80							11.80
11.90							11.90
12.00							12.00
12.10							12.10
12.20							12.20
12.30							12.30
12.40							12.40
12.50							12.50
12.60							12.60
12.70							12.70
12.80							12.80
12.90							12.90
13.00							13.00
13.10							13.10
13.20							13.20
13.30							13.30
13.40							13.40
13.50							13.50
13.60							13.60
13.70							13.70
13.80							13.80
13.90							13.90
14.00							14.00
14.10							14.10
14.20							14.20
14.30							14.30
14.40							14.40
14.50							14.50
14.60							14.60
14.70							14.70
14.80							14.80
14.90							14.90

t	SKEWNESS .0	.1	.2	.3	4	.5	t
-9.90							-9.90
-9.80							-9.80
-9.70							-9.70
-9.60							-9.60
-9.50							-9.50
-9.40							-9.40
-9.30							-9.30
-9.20							-9.20
-9.10							-9.10
-9.00							-9.00
-8.90							-8.90
-8.80							-8.80
-8.70							-8.70
-8.60							-8.60
-8.50							-8.50
-8.40							-8.40
-8.30							-8.30
-8.20							-8.20
-8.10							-8.10
-8.00							-8.00
-7.90							-7.90
-7.80							-7.80
-7.70							-7.70
-7.60							-7.60
-7.50							-7.50
-7.40							-7.40
-7.30							-7.30
-7.20							-7.20
-7.10							-7.10
-7.00							-7.00
-6.90							-6.90
-6.80							-6.80
-6.70							-6.70
-6.60							-6.60
-6.50							-6.50
-6.40							-6.40
-6.30							-6.30
-6.20							-6.20
-6.10							-6.10
-6.00							-6.00
-5.90							-5.90
-5.80	.000001						-5.80
-5.70	.000001						-5.70
-5.60	.000002						-5.60
-5.50	.000003						-5.50
-5.40	.000005						-5.40
-5.30	.000009	.000001					-5.30
-5.20	.000014	.000002					-5.20
-5.10	.000022	.000003					-5.10
-5.00	.000036	.000006					-5.00

	SKEWNESS						
t	.6	.7	.8	.9	1.0	1.1	t
-9.90							-9.90
-9.80							-9.80
-9.70							-9.70
-9.60							-9.60
-9.50							-9.50
-9.40							-9.40
-9.30							-9.30
-9.20							-9.20
-9.10							-9.10
-9.00							-9.00
-8.90							-8.90
-8.80							-8.80
-8.70							-8.70
-8.60							-8.60
-8.50							-8.50
-8.40							-8.40
-8.30							-8.30
-8.20							-8.20
-8.10							-8.10
-8.00							-8.00
-7.90							-7.90
-7.80							-7.80
-7.70							-7.70
-7.60							-7.60
-7.50							-7.50
-7.40							-7.40
-7.30							-7.30
-7.20							-7.20
-7.10							-7.10
-7.00							-7.00
-6.90							-6.90
-6.80							-6.80
-6.70							-6.70
-6.60							-6.60
-6.50							-6.50
-6.40							-6.40
-6.30							-6.30
-6.20							-6.20
-6.10							-6.10
-6.00							-6.00
-5.90							-5.90
-5.80							-5.80
-5.70							-5.70
-5.60							-5.60
-5.50							-5.50
-5.40							-5.40
-5.30							-5.30
-5.20							-5.20
-5.10							-5.10
-5.00							-5.00

t	SKEWNESS						t
	.0	.1	.2	.3	.4	.5	
–4.90	.000056	.000011	.000001				–4.90
–4.80	.000087	.000020	.000002				–4.80
–4.70	.000134	.000036	.000004				–4.70
–4.60	.000204	.000061	.000008				–4.60
–4.50	.000308	.000103	.000017				–4.50
–4.40	.000458	.000171	.000034	.000001			–4.40
–4.30	.000674	.000279	.000066	.000004			–4.30
–4.20	.000981	.000446	.000124	.000011			–4.20
–4.10	.001411	.000702	.000227	.000029			–4.10
–4.00	.002007	.001085	.000404	.000068	.000001		–4.00
–3.90	.002823	.001650	.000698	.000151	.000005		–3.90
–3.80	.003924	.002465	.001174	.000318	.000020		–3.80
–3.70	.005390	.003622	.001921	.000638	.000065		–3.70
–3.60	.007318	.005233	.003063	.001216	.000190		–3.60
–3.50	.009818	.007435	.004759	.002213	.000493	.000008	–3.50
–3.40	.013012	.010387	.007210	.003855	.001157	.000056	–3.40
–3.30	.017036	.014272	.010658	.006445	.002482	.000259	–3.30
–3.20	.022029	.019282	.015376	.010360	.004921	.000914	–3.20
–3.10	.028127	.025617	.021661	.016039	.009087	.002622	–3.10
–3.00	.035455	.033464	.029805	.023959	.015732	.006377	–3.00
–2.90	.044108	.042977	.040072	.034578	.025682	.013553	–2.90
–2.80	.054142	.054254	.052649	.048276	.039717	.025751	–2.80
–2.70	.065547	.067309	.067607	.065269	.058421	.044473	–2.70
–2.60	.078238	.082041	.084853	.085530	.082005	.070719	–2.60
–2.50	.092024	.098208	.104081	.108701	.110147	.104574	–2.50
–2.40	.106598	.115397	.124739	.134041	.141874	.144908	–2.40
–2.30	.121523	.133016	.146010	.160398	.175522	.189277	–2.30
–2.20	.136222	.150286	.166813	.186228	.208791	.234060	–2.20
–2.10	.149984	.166250	.185835	.209664	.238900	.274831	–2.10
–2.00	.161973	.179808	.201589	.228635	.262822	.306883	–2.00
–1.90	.171257	.189761	.212507	.241024	.277610	.325833	–1.90
–1.80	.176848	.194883	.217043	.244859	.280668	.328187	–1.80
–1.70	.177753	.193997	.213810	.238508	.270098	.311788	–1.70
–1.60	.173036	.186074	.201707	.220856	.244914	.276085	–1.60
–1.50	.161897	.170327	.180042	.191456	.205187	.222196	–1.50
–1.40	.143738	.146302	.148634	.150619	.152083	.152768	–1.40
–1.30	.118244	.113955	.107884	.099440	.087784	.071682	–1.30
–1.20	.085442	.073710	.058793	.039758	.015318	–.016353	–1.20
–1.10	.045749	.026480	.002942	–.025961	–.061696	–.106262	–1.10
–1.00	.000000	–.026345	–.057582	–.094785	–.139336	–.193043	–1.00
– .90	–.050556	–.082945	–.120304	–.163523	–.213693	–.272169	– .90
– .80	–.104289	–.141156	–.182513	–.228950	–.281149	–.339887	– .80
– .70	–.159250	–.198588	–.241433	–.288031	–.338624	–.393425	– .70
– .60	–.213264	–.252765	–.294407	–.338118	–.383756	–.431081	– .60
– .50	–.264049	–.301276	–.339069	–.377114	–.415005	–.452220	– .50
– .40	–.309347	–.341929	–.373489	–.403584	–.431690	–.457186	– .40
– .30	–.347063	–.372891	–.396290	–.416811	–.433951	–.447153	– .30
– .20	–.375401	–.392803	–.406717	–.416794	–.422664	–.423938	– .20
– .10	–.392983	–.400862	–.404661	–.404202	–.399307	–.389805	– .10
.00	–.398942	–.396865	–.390638	–.380276	–.365806	–.347261	.00

t	SKEWNESS						t
	.6	.7	.8	.9	1.0	1.1	
–4.90							–4.90
–4.80							–4.80
–4.70							–4.70
–4.60							–4.60
–4.50							–4.50
–4.40							–4.40
–4.30							–4.30
–4.20							–4.20
–4.10							–4.10
–4.00							–4.00
–3.90							–3.90
–3.80							–3.80
–3.70							–3.70
–3.60							–3.60
–3.50							–3.50
–3.40							–3.40
–3.30							–3.30
–3.20	.000008						–3.20
–3.10	.000036						–3.10
–3.00	.000442						–3.00
–2.90	.002377						–2.90
–2.80	.008378						–2.80
–2.70	.021975	.001288					–2.70
–2.60	.046701	.011088					–2.60
–2.50	.084682	.040658					–2.50
–2.40	.135492	.097040	.014364				–2.40
–2.30	.195696	.178156	.093291				–2.30
–2.20	.259189	.273180	.235123	.015680			–2.20
–2.10	.318205	.366237	.398395	.290543			–2.10
–2.00	.364653	.440957	.537617	.618006			–2.00
–1.90	.391464	.484203	.620767	.826786	1.056709		–1.90
–1.80	.393656	.488248	.633769	.878055	1.344215	2.171541	–1.80
–1.70	.368970	.451364	.578139	.792281	1.211776	2.318708	–1.70
–1.60	.318022	.377196	.466054	.611436	.881883	1.511052	–1.60
–1.50	.244037	.273391	.315228	.379697	.490506	.715683	–1.50
–1.40	.152272	.149950	.144709	.134579	.115659	.079111	–1.40
–1.30	.049275	.017638	–.028037	–.096151	–.202538	–.380314	–1.30
–1.20	–.057908	–.113328	–.188820	–294517	–.447941	–.681730	–1.20
–1.10	–.162444	–.234218	–.327426	–.450951	–.618879	–.854725	–1.10
–1.00	–.258300	–.338340	–.437616	–.562403	–.721788	–.929360	–1.00
– .90	–.340641	–.421241	–.516683	–.630449	–.767053	–.932389	– .90
– .80	–.406052	–.480646	–.564776	–.659627	–.766385	–.886072	– .80
– .70	–.452596	–.516205	–.584160	–.656102	–.731248	–.808124	– .70
– .60	–.479724	–.529131	–.578496	–.626664	–.671992	–.712150	– .60
– .50	–.488097	–.521789	–.552221	–.578022	–.597445	–.608256	– .50
– .40	–.479345	–.497304	–.510042	–.516352	–.514799	–.503678	– .40
– .30	–.455795	–.459186	–.456554	–.447036	–.429670	–.403375	– .30
– .20	–.420207	–.411038	–.395979	–.374550	–.346246	–.310536	– .20
– .10	–.375528	–.356312	–.332002	–.302445	–.267495	–.227013	– .10
.00	–.324689	–.298144	–.267691	–.233404	–.195367	–.153671	.00

	SKEWNESS						
t	.0	.1	.2	.3	.4	.5	t
.00	-.398942	-.396865	-.390638	-.380276	-.365806	-.347261	.00
.10	-.392983	-.381208	-.365722	-.346712	-.324368	-.298881	.10
.20	-.375401	-.354845	-.331450	-.305516	-.277327	-.247150	.20
.30	-.347063	-.319212	-.289703	-.258865	-.226991	-.194347	.30
.40	-.309347	-.276121	-.242569	-.208963	-.175528	-.142459	.40
.50	-.264049	-.227632	-.192211	-.157924	-.124871	-.093127	.50
.60	-.213264	-.175927	-.140741	-.107668	-.076652	-.047627	.60
.70	-.159250	-.123170	-.090112	-.059849	-.032168	-.006874	.70
.80	-.104289	-.071396	-.042031	-.015803	.007628	.028559	.80
.90	-.050556	-.022408	.002100	.023470	.042119	.058401	.90
1.00	.000000	.022298	.041221	.057307	.070994	.082638	1.00
1.10	.045749	.061582	.074619	.085362	.094207	.101468	1.10
1.20	.085442	.094674	.101920	.107574	.111937	.115246	1.20
1.30	.118244	.121175	.123055	.124117	.124535	.124440	1.30
1.40	.143738	.141025	.138219	.135360	.132477	.129592	1.40
1.50	.161897	.154462	.147816	.141807	.136322	.131273	1.50
1.60	.173036	.161963	.152411	.144058	.136668	.130063	1.60
1.70	.177753	.164185	.152670	.142759	.134125	.126521	1.70
1.80	.176848	.161901	.149315	.138570	.129283	.121170	1.80
1.90	.171257	.155944	.143079	.132127	.122694	.114484	1.90
2.00	.161973	.147151	.134668	.124028	.114859	.106882	2.00
2.10	.149984	.136328	.122839	.114809	.106219	.098724	2.10
2.20	.136222	.124207	.113878	.104940	.097149	.090313	2.20
2.30	.121523	.111433	.102586	.094814	.087962	.081894	2.30
2.40	.106598	.098547	.091279	.084754	.078907	.073662	2.40
2.50	.092024	.085981	.080285	.075012	.070177	.065763	2.50
2.60	.078238	.074064	.069851	.065773	.061913	.058305	2.60
2.70	.065547	.063024	.060152	.057169	.054212	.051357	2.70
2.80	.054142	.053008	.051295	.049280	.047134	.044963	2.80
2.90	.044108	.044085	.043335	.042146	.040707	.039139	2.90
3.00	.035455	.036269	.036285	.035776	.034933	.033885	3.00
3.10	.028127	.029527	.030122	.030152	.029797	.029186	3.10
3.20	.022029	.023795	.024799	.025238	.025271	.025017	3.20
3.30	.017036	.018986	.020254	.020987	.021315	.021343	3.30
3.40	.013012	.015004	.016415	.017341	.017883	.018129	3.40
3.50	.009818	.011746	.013204	.014242	.014928	.015334	3.50
3.60	.007318	.009111	.010544	.011628	.012402	.012917	3.60
3.70	.005390	.007004	.008361	.009440	.010254	.010839	3.70
3.80	.003924	.005336	.006585	.007621	.008441	.009061	3.80
3.90	.002823	.004031	.005151	.006120	.006918	.007548	3.90
4.00	.002007	.003019	.004003	.004890	.005646	.006266	4.00
4.10	.001411	.002242	.003091	.003887	.004590	.005184	4.10
4.20	.000981	.001652	.002372	.003075	.003716	.004275	4.20
4.30	.000674	.001207	.001810	.002421	.002997	.003515	4.30
4.40	.000458	.000875	.001372	.001897	.002409	.002881	4.40
4.50	.000308	.000629	.001034	.001480	.001929	.002355	4.50
4.60	.000204	.000449	.000775	.001150	.001539	.001919	4.60
4.70	.000134	.000318	.000578	.000889	.001224	.001560	4.70
4.80	.000087	.000224	.000428	.000685	.000970	.001265	4.80
4.90	.000056	.000156	.000316	.000525	.000767	.001023	4.90

t	SKEWNESS						t
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.20	-.215242	-.181842	-.147179	-.111467	-.074908	-.037693	.20
.30	-.161173	-.127682	-.094065	-.060496	-.027127	.005906	.30
.40	-.109915	-.078031	-.046919	-.016670	.012641	.040955	.40
.50	-.062736	-.033730	-.006120	.020094	.044920	.068375	.50
.60	-.020517	.004757	.028276	.050122	.070377	.089119	.60
.70	.016219	.037278	.056460	.073908	.089753	.104118	.70
.80	.047251	.063932	.078805	.092047	.103816	.114254	.80
.90	.072612	.085008	.095804	.105187	.113319	.120340	.90
1.00	.092535	.100929	.108025	.113993	.118980	.123110	1.00
1.10	.107400	.112210	.116068	.119113	.121463	.123215	1.10
1.20	.117687	.119411	.120537	.121163	.121369	.121222	1.20
1.30	.123938	.123108	.122015	.120710	.119234	.117620	1.30
1.40	.126718	.123868	.121049	.118266	.115524	.112825	1.40
1.50	.126593	.122229	.118138	.114285	.110642	.107185	1.50
1.60	.124106	.118689	.113730	.109161	.104928	.100987	1.60
1.70	.119760	.113697	.108217	.103232	.098668	.094466	1.70
1.80	.114013	.107646	.101936	.096780	.092094	.087811	1.80
1.90	.107270	.100878	.095170	.090039	.085395	.081170	1.90
2.00	.099878	.093681	.088156	.083198	.078722	.074658	2.00
2.10	.092132	.086292	.081084	.076409	.072189	.068360	2.10
2.20	.084273	.078905	.074104	.069787	.065884	.062340	2.20
2.30	.076495	.071668	.067332	.063418	.059870	.056640	2.30
2.40	.068947	.064697	.060854	.057366	.054190	.051288	2.40
2.50	.061741	.058075	.054730	.051672	.048870	.046297	2.50
2.60	.054956	.051859	.049000	.046362	.043925	.041672	2.60
2.70	.048642	.046084	.043686	.041446	.039357	.037410	2.70
2.80	.042828	.040765	.038796	.036927	.035163	.033501	2.80
2.90	.037523	.035908	.034327	.032797	.031330	.029931	2.90
3.00	.032722	.031503	.030268	.029043	.027845	.026685	3.00
3.10	.028411	.027535	.026603	.025648	.024689	.023743	3.10
3.20	.024565	.023980	.023311	.022590	.021843	.021085	3.20
3.30	.021156	.020815	.020367	.019848	.019284	.018692	3.30
3.40	.018152	.018009	.017747	.017399	.016991	.016543	3.40
3.50	.015519	.015535	.015424	.015218	.014942	.014618	3.50
3.60	.013222	.013362	.013372	.013283	.013118	.012897	3.60
3.70	.011229	.011461	.011566	.011570	.011497	.011363	3.70
3.80	.009506	.009804	.009981	.010060	.010060	.009998	3.80
3.90	.008024	.008366	.008596	.008731	.008789	.008785	3.90
4.00	.006753	.007122	.007387	.007564	.007668	.007711	4.00
4.10	.005668	.006049	.006336	.006543	.006680	.006760	4.10
4.20	.004745	.005126	.005425	.005651	.005813	.005919	4.20
4.30	.003961	.004334	.004636	.004873	.005051	.005178	4.30
4.40	.003299	.003658	.003955	.004196	.004384	.004525	4.40
4.50	.002742	.003080	.003369	.003608	.003800	.003950	4.50
4.60	.002273	.002589	.002865	.003098	.003291	.003446	4.60
4.70	.001880	.002173	.002433	.002657	.002847	.003003	4.70
4.80	.001552	.001820	.002062	.002276	.002460	.002614	4.80
4.90	.001278	.001522	.001746	.001947	.002124	.002274	4.90

t	SKEWNESS .0	.1	.2	.3	.4	.5	t
5.00	.000036	.000108	.000232	.000401	.000604	.000825	5.00
5.10	.000022	.000074	.000169	.000305	.000474	.000664	5.10
5.20	.000014	.000051	.000123	.000232	.000371	.000533	5.20
5.30	.000009	.000034	.000089	.000175	.000290	.000427	5.30
5.40	.000005	.000023	.000064	.000132	.000226	.000341	5.40
5.50	.000003	.000015	.000046	.000099	.000175	.000272	5.50
5.60	.000002	.000010	.000032	.000074	.000136	.000216	5.60
5.70	.000001	.000007	.000023	.000055	.000105	.000172	5.70
5.80	.000001	.000004	.000016	.000041	.000081	.000136	5.80
5.90		.000003	.000011	.000030	.000062	.000108	5.90
6.00		.000002	.000008	.000022	.000048	.000085	6.00
6.10		.000001	.000006	.000016	.000036	.000067	6.10
6.20		.000001	.000004	.000012	.000028	.000053	6.20
6.30			.000003	.000009	.000021	.000041	6.30
6.40			.000002	.000006	.000016	.000032	6.40
6.50			.000001	.000005	.000012	.000025	6.50
6.60			.000001	.000003	.000009	.000020	6.60
6.70			.000001	.000002	.000007	.000015	6.70
6.80				.000002	.000005	.000012	6.80
6.90				.000001	.000004	.000009	6.90
7.00				.000001	.000003	.000007	7.00
7.10				.000001	.000002	.000006	7.10
7.20					.000002	.000004	7.20
7.30					.000001	.000003	7.30
7.40					.000001	.000003	7.40
7.50					.000001	.000002	7.50
7.60					.000001	.000002	7.60
7.70						.000001	7.70
7.80						.000001	7.80
7.90						.000001	7.90
8.00						.000001	8.00
8.10							8.10
8.20							8.20
8.30							8.30
8.40							8.40
8.50							8.50
8.60							8.60
8.70							8.70
8.80							8.80
8.90							8.90
9.00							9.00
9.10							9.10
9.20							9.20
9.30							9.30
9.40							9.40
9.50							9.50
9.60							9.60
9.70							9.70
9.80							9.80
9.90							9.90

t	SKEWNESS						t
	.6	.7	.8	.9	1.0	1.1	
5.00	.001051	.001270	.001476	.001664	.001832	.001977	5.00
5.10	.000862	.001059	.001247	.001421	.001578	.001717	5.10
5.20	.000706	.000881	.001051	.001212	.001359	.001491	5.20
5.30	.000577	.000732	.000885	.001032	.001169	.001293	5.30
5.40	.000471	.000607	.000745	.000878	.001004	.001121	5.40
5.50	.000383	.000503	.000626	.000747	.000863	.000971	5.50
5.60	.000312	.000416	.000525	.000634	.000740	.000840	5.60
5.70	.000253	.000344	.000440	.000538	.000635	.000727	5.70
5.80	.000205	.000284	.000369	.000456	.000544	.000629	5.80
5.90	.000166	.000234	.000309	.000387	.000466	.000543	5.90
6.00	.000134	.000193	.000258	.000327	.000399	.000469	6.00
6.10	.000108	.000158	.000215	.000277	.000341	.000405	6.10
6.20	.000087	.000130	.000180	.000234	.000291	.000349	6.20
6.30	.000070	.000107	.000150	.000198	.000249	.000301	6.30
6.40	.000056	.000087	.000125	.000167	.000212	.000260	6.40
6.50	.000045	.000072	.000104	.000141	.000181	.000224	6.50
6.60	.000036	.000058	.000086	.000119	.000154	.000193	6.60
6.70	.000029	.000048	.000072	.000100	.000132	.000166	6.70
6.80	.000023	.000039	.000059	.000084	.000112	.000143	6.80
6.90	.000018	.000032	.000049	.000071	.000095	.000123	6.90
7.00	.000015	.000026	.000041	.000059	.000081	.000105	7.00
7.10	.000012	.000021	.000034	.000050	.000069	.000091	7.10
7.20	.000009	.000017	.000028	.000042	.000059	.000078	7.20
7.30	.000007	.000014	.000023	.000035	.000050	.000067	7.30
7.40	.000006	.000011	.000019	.000029	.000042	.000057	7.40
7.50	.000005	.000009	.000016	.000025	.000036	.000049	7.50
7.60	.000004	.000007	.000013	.000021	.000030	.000042	7.60
7.70	.000003	.000006	.000011	.000017	.000026	.000036	7.70
7.80	.000002	.000005	.000009	.000015	.000022	.000031	7.80
7.90	.000002	.000004	.000007	.000012	.000019	.000027	7.90
8.00	.000001	.000003	.000006	.000010	.000016	.000023	8.00
8.10	.000001	.000003	.000005	.000008	.000013	.000019	8.10
8.20	.000001	.000002	.000004	.000007	.000011	.000017	8.20
8.30	.000001	.000002	.000003	.000006	.000010	.000014	8.30
8.40	.000001	.000001	.000003	.000005	.000008	.000012	8.40
8.50		.000001	.000002	.000004	.000007	.000010	8.50
8.60		.000001	.000002	.000003	.000006	.000009	8.60
8.70		.000001	.000002	.000003	.000005	.000008	8.70
8.80		.000001	.000001	.000002	.000004	.000007	8.80
8.90			.000001	.000002	.000003	.000006	8.90
9.00			.000001	.000002	.000003	.000005	9.00
9.10			.000001	.000001	.000002	.000004	9.10
9.20			.000001	.000001	.000002	.000003	9.20
9.30				.000001	.000002	.000003	9.30
9.40				.000001	.000001	.000003	9.40
9.50				.000001	.000001	.000002	9.50
9.60				.000001	.000001	.000002	9.60
9.70					.000001	.000002	9.70
9.80					.000001	.000001	9.80
9.90					.000001	.000001	9.90

	SKEWNESS						
t	.0	.1	.2	.3	.4	.5	t
10.00							10.00
10.10							10.10
10.20							10.20
10.30							10.30
10.40							10.40
10.50							10.50
10.60							10.60
10.70							10.70
10.80							10.80
10.90							10.90
11.00							11.00
11.10							11.10
11.20							11.20
11.30							11.30
11.40							11.40
11.50							11.50
11.60							11.60
11.70							11.70
11.80							11.80
11.90							11.90
12.00							12.00
12.10							12.10
12.20							12.20
12.30							12.30
12.40							12.40
12.50							12.50
12.60							12.60
12.70							12.70
12.80							12.80
12.90							12.90
13.00							13.00
13.10							13.10
13.20							13.20
13.30							13.30
13.40							13.40
13.50							13.50
13.60							13.60
13.70							13.70
13.80							13.80
13.90							13.90
14.00							14.00
14.10							14.10
14.20							14.20
14.30							14.30
14.40							14.40
14.50							14.50
14.60							14.60
14.70							14.70
14.80							14.80
14.90							14.90

t	SKEWNESS .6	.7	.8	.9	1.0	1.1	t
10.00					.000001	.000001	10.00
10.10						.000001	10.10
10.20						.000001	10.20
10.30						.000001	10.30
10.40						.000001	10.40
10.50							10.50
10.60							10.60
10.70							10.70
10.80							10.80
10.90							10.90
11.00							11.00
11.10							11.10
11.20							11.20
11.30							11.30
11.40							11.40
11.50							11.50
11.60							11.60
11.70							11.70
11.80							11.80
11.90							11.90
12.00							12.00
12.10							12.10
12.20							12.20
12.30							12.30
12.40							12.40
12.50							12.50
12.60							12.60
12.70							12.70
12.80							12.80
12.90							12.90
13.00							13.00
13.10							13.10
13.20							13.20
13.30							13.30
13.40							13.40
13.50							13.50
13.60							13.60
13.70							13.70
13.80							13.80
13.90							13.90
14.00							14.00
14.10							14.10
14.20							14.20
14.30							14.30
14.40							14.40
14.50							14.50
14.60							14.60
14.70							14.70
14.80							14.80
14.90							14.90

	SKEWNESS						
t	.0	.1	.2	.3	.4	.5	t
–9.90							–9.90
–9.80							–9.80
–9.70							–9.70
–9.60							–9.60
–9.50							–9.50
–9.40							–9.40
–9.30							–9.30
–9.20							–9.20
–9.10							–9.10
–9.00							–9.00
–8.90							–8.90
–8.80							–8.80
–8.70							–8.70
–8.60							–8.60
–8.50							–8.50
–8.40							–8.40
–8.30							–8.30
–8.20							–8.20
–8.10							–8.10
–8.00							–8.00
–7.90							–7.90
–7.80							–7.80
–7.70							–7.70
–7.60							–7.60
–7.50							–7.50
–7.40							–7.40
–7.30							–7.30
–7.20							–7.20
–7.10							–7.10
–7.00							–7.00
–6.90							–6.90
–6.80							–6.80
–6.70							–6.70
–6.60							–6.60
–6.50							–6.50
–6.40							–6.40
–6.30							–6.30
–6.20							–6.20
–6.10							–6.10
–6.00	.000001						–6.00
–5.90	.000002						–5.90
–5.80	.000004						–5.80
–5.70	.000006						–5.70
–5.60	.000010						–5.60
–5.50	.000016	.000002					–5.50
–5.40	.000026	.000003					–5.40
–5.30	.000042	.000006					–5.30
–5.20	.000067	.000012					–5.20
–5.10	.000105	.000021					–5.10
–5.00	.000164	.000038	.000003				–5.00

	SKEWNESS						
t	.6	.7	.8	.9	1.0	1.1	t
-9.90							-9.90
-9.80							-9.80
-9.70							-9.70
-9.60							-9.60
-9.50							-9.50
-9.40							-9.40
-9.30							-9.30
-9.20							-9.20
-9.10							-9.10
-9.00							-9.00
-8.90							-8.90
-8.80							-8.80
-8.70							-8.70
-8.60							-8.60
-8.50							-8.50
-8.40							-8.40
-8.30							-8.30
-8.20							-8.20
-8.10							-8.10
-8.00							-8.00
-7.90							-7.90
-7.80							-7.80
-7.70							-7.70
-7.60							-7.60
-7.50							-7.50
-7.40							-7.40
-7.30							-7.30
-7.20							-7.20
-7.10							-7.10
-7.00							-7.00
-6.90							-6.90
-6.80							-6.80
-6.70							-6.70
-6.60							-6.60
-6.50							-6.50
-6.40							-6.40
-6.30							-6.30
-6.20							-6.20
-6.10							-6.10
-6.00							-6.00
-5.90							-5.90
-5.80							-5.80
-5.70							-5.70
-5.60							-5.60
-5.50							-5.50
-5.40							-5.40
-5.30							-5.30
-5.20							-5.20
-5.10							-5.10
-5.00							-5.00

t	SKEWNESS						t
	.0	.1	.2	.3	.4	.5	
-4.90	.000251	.000067	.000006				-4.90
-4.80	.000381	.000116	.000014				-4.80
-4.70	.000572	.000196	.000029				-4.70
-4.60	.000847	.000326	.000060				-4.60
-4.50	.001241	.000531	.000120				-4.50
-4.40	.001795	.000850	.000230	.000016			-4.40
-4.30	.002567	.001336	.000427	.000043			-4.30
-4.20	.003624	.002060	.000768	.000109			-4.20
-4.10	.005054	.003119	.001340	.000255			-4.10
-4.00	.006959	.004637	.002267	.000563	.000018		-4.00
-3.90	.009460	.006766	.003724	.001168	.000073		-3.90
-3.80	.012692	.009694	.005942	.002293	.000251		-3.80
-3.70	.016801	.013632	.009213	.004267	.000740		-3.70
-3.60	.021940	.018816	.013889	.007556	.001917		-3.60
-3.50	.028253	.025486	.020368	.012760	.004442	.000178	-3.50
-3.40	.035863	.033867	.029060	.020595	.009325	.000949	-3.40
-3.30	.044851	.044140	.040347	.031834	.017934	.003585	-3.30
-3.20	.055235	.056401	.054515	.047196	.031871	.010507	-3.20
-3.10	.066940	.070617	.071673	.067197	.052709	.025309	-3.10
-3.00	.079773	.086578	.091666	.091968	.081588	.052096	-3.00
-2.90	.093389	.103848	.113982	.121059	.118728	.094158	-2.90
-2.80	.107270	.121725	.137680	.153273	.162975	.152364	-2.80
-2.70	.120705	.139216	.161342	.186568	.211485	.223811	-2.70
-2.60	.132787	.155031	.183074	.218055	.263099	.301307	-2.60
-2.50	.142417	.167610	.200569	.244130	.301543	.373931	-2.50
-2.40	.148341	.175189	.211232	.260747	.330213	.428654	-2.40
-2.30	.149198	.175896	.212378	.263811	.338890	.452620	-2.30
-2.20	.143601	.167894	.201481	.249661	.321821	.435578	-2.20
-2.10	.130235	.149545	.176464	.215583	.275271	.371893	-2.10
-2.00	.107982	.119605	.135996	.160271	.198270	.261753	-2.00
-1.90	.076048	.077409	.079756	.084193	.093031	.111349	-1.90
-1.80	.034106	.023049	.008637	-.010236	-.035083	-.067918	-1.80
-1.70	-.017587	-.042496	-.075148	-.118678	-.177972	-.260996	-1.70
-1.60	-.078088	-.117286	-.168089	-.235127	-.325592	-.451173	-1.60
-1.50	-.145707	-.198427	-.265531	-.352360	-.466992	-.622148	-1.50
-1.40	-.218003	-.282164	-.361956	-.462545	-.591433	-.759871	-1.40
-1.30	-.291841	-.364068	-.451376	-.557937	-.689459	-.853914	-1.30
-1.20	-.363516	-.439306	-.527799	-.631571	-.753783	-.898261	-1.20
-1.10	-.428951	-.502977	-.561923	-.677890	-.779897	-.891507	-1.10
-1.00	-.483941	-.550479	-.531037	-.693231	-.766350	-.836520	-1.00
- .90	-.524454	-.577893	-.629285	-.676116	-.714713	-.739705	- .90
- .80	-.546938	-.582320	-.610227	-.627340	-.629237	-.610023	- .80
- .70	-.548630	-.562167	-.563714	-.549840	-.516310	-.457901	- .70
- .60	-.527828	-.517322	-.491807	-.448387	-.383756	-.294180	- .60
- .50	-.484090	-.449217	-.398197	-.329133	-.240099	-.129206	- .50
- .40	-.418355	-.360770	-.287898	-.199071	-.093846	.027898	- .40
- .30	-.332952	-.256192	-.166859	-.065468	.047137	.169710	- .30
- .20	-.231497	-.140701	-.041502	.064673	.176110	.290779	- .20
- .10	-.118689	-.020144	.081750	.185082	.287793	.387675	- .10
.00	.000000	.099416	.196913	.290583	.378529	.458881	.00

t	SKEWNESS						t
	.6	.7	.8	.9	1.0	1.1	
-4.90							-4.90
-4.80							-4.80
-4.70							-4.70
-4.60							-4.60
-4.50							-4.50
-4.40							-4.40
-4.30							-4.30
-4.20							-4.20
-4.10							-4.10
-4.00							-4.00
-3.90							-3.90
-3.80							-3.80
-3.70							-3.70
-3.60							-3.60
-3.50							-3.50
-3.40							-3.40
-3.30							-3.30
-3.20	.000428						-3.20
-3.10	.001108						-3.10
-3.00	.008908						-3.00
-2.90	.034461						-2.90
-2.80	.091644						-2.80
-2.70	.186439	.037335					-2.70
-2.60	.[illegible]11828	.179031					-2.60
-2.50	.[illegible]47156	.424944					-2.50
-2.40	.563261	.699116	.412736				-2.40
-2.30	.630460	.904484	1.154771				-2.30
-2.20	.626233	.968743	1.606671	1.308533			-2.20
-2.10	.540369	.864479	1.580220	3.463876			-2.10
-2.00	.376624	.607660	1.150250	2.823381			-2.00
-1.90	.151178	.244293	.489790	1.298782	6.008392		-1.90
-1.80	-.111123	-.166161	-.226346	-.233321	.314604	26.296986	-1.80
-1.70	-.381520	-.565200	-.865030	-1.409161	-2.599172	-6.749225	-1.70
-1.60	-.631967	-.905133	-1.346380	-2.132746	-3.776760	-8.464687	-1.60
-1.50	-.838850	-1.154030	-1.637910	-2.439060	-3.924047	-7.246598	-1.50
-1.40	-.985394	-1.296626	-1.743088	-2.417049	-3.508310	-5.465547	-1.40
-1.30	-1.062686	-1.332423	-1.688081	-2.167842	-2.830276	-3.758660	-1.30
-1.20	-1.069482	-1.272258	-1.510562	-1.783859	-2.076035	-2.320434	-1.20
-1.10	-1.011123	-1.134368	-1.251236	-1.340054	-1.354128	-1.190123	-1.10
-1.00	-.897900	-.940660	-.948168	-.891892	-.721788	-.347336	-1.00
- .90	-.743216	-.713610	-.633538	-.476810	-.203287	.249654	- .90
- .80	-.561803	-.473971	-.332221	-.117191	.197402	.647237	- .80
- .70	-.368188	-.239300	-.061661	.176251	.488027	.889156	- .70
- .60	-.175509	-.023216	.167459	.401253	.682370	1.013589	- .60
- .50	.005280	.164776	.350128	.561253	.796593	1.052338	- .50
- .40	.165834	.319034	.485754	.663162	.846984	1.031034	- .40
- .30	.300524	.437271	.576943	.715691	.848660	.969782	- .30
- .20	.406299	.519904	.628401	.728128	.814903	.883974	- .20
- .10	.482370	.569371	.646021	.709516	.756908	.785099	- .10
.00	.529797	.589476	.636159	.668143	.683784	.681502	.00

t	SKEWNESS						t
	.0	.1	.2	.3	.4	.5	
.00	.000000	.099416	.196913	.290583	.378529	.458881	.00
.10	.118689	.212123	.298828	.377333	.446308	.504560	.10
.20	.231497	.312723	.383442	.442932	.490655	.526245	.20
.30	.332952	.396892	.447991	.486405	.512433	.526493	.30
.40	.418355	.461478	.491064	.508096	.513583	.508539	.40
.50	.484090	.504634	.512563	.509471	.496829	.475980	.50
.60	.527828	.525849	.513571	.492878	.465389	.432494	.60
.70	.548630	.525878	.496156	.461276	.422704	.381621	.70
.80	.546938	.506577	.463119	.417974	.372202	.326592	.80
.90	.524454	.470682	.417735	.366379	.317109	.270225	.90
1.00	.483941	.421543	.363493	.309789	.260310	.214859	1.00
1.10	.428951	.362844	.303855	.251224	.204259	.162335	1.10
1.20	.363516	.298335	.242061	.193306	.150934	.114009	1.20
1.30	.291841	.231592	.180974	.138185	.101825	.070791	1.30
1.40	.218003	.165822	.122976	.087512	.057959	.033194	1.40
1.50	.145707	.103725	.069917	.042440	.019940	.001404	1.50
1.60	.078088	.047402	.023097	.003664	−.011988	−.024665	1.60
1.70	.017587	−.001667	−.016702	−.028531	−.037884	−.045300	1.70
1.80	−.034106	−.042615	−.049166	−.054198	−.058037	−.060934	1.80
1.90	−.076048	−.075129	−.074377	−.073658	−.072905	−.072092	1.90
2.00	−.107982	−.099370	−.092732	−.087429	−.083057	−.079351	2.00
2.10	−.130235	−.115877	−.101088	−.096161	−.089123	−.083300	2.10
2.20	−.143601	−.125462	−.111534	−.100572	−.091752	−.084515	2.20
2.30	−.149198	−.129110	−.113618	−.101402	−.091579	−.083537	2.30
2.40	−.148341	−.127891	−.111985	−.099375	−.089202	−.080863	2.40
2.50	−.142417	−.122878	−.107484	−.095167	−.085165	−.076931	2.50
2.60	−.132787	−.115092	−.100896	−.089387	−.079949	−.072122	2.60
2.70	−.120705	−.105455	−.092916	−.082565	−.073964	−.066758	2.70
2.80	−.107270	−.094763	−.084134	−.075151	−.067554	−.061105	2.80
2.90	−.093389	−.083674	−.075038	−.067508	−.060996	−.055373	2.90
3.00	−.079773	−.072706	−.066010	−.059925	−.054507	−.049728	3.00
3.10	−.066940	−.062242	−.057337	−.052617	−.048251	−.044293	3.10
3.20	−.055235	−.052547	−.049222	−.045738	−.042343	−.039155	3.20
3.30	−.044851	−.043785	−.041793	−.039388	−.036861	−.034374	3.30
3.40	−.035863	−.036031	−.035120	−.033623	−.031849	−.029982	3.40
3.50	−.028253	−.029301	−.029225	−.028467	−.027326	−.025994	3.50
3.60	−.021940	−.023558	−.024094	−.023915	−.023291	−.022409	3.60
3.70	−.016801	−.018734	−.019688	−.019942	−.019729	−.019217	3.70
3.80	−.012692	−.014741	−.015951	−.016514	−.016613	−.016396	3.80
3.90	−.009460	−.011481	−.012818	−.013583	−.013911	−.013924	3.90
4.00	−.006959	−.008854	−.010220	−.011101	−.011586	−.011771	4.00
4.10	−.005054	−.006762	−.008087	−.009017	−.009601	−.009908	4.10
4.20	−.003624	−.005116	−.006352	−.007281	−.007917	−.008306	4.20
4.30	−.002567	−.003835	−.004954	−.005845	−.006498	−.006936	4.30
4.40	−.001795	−.002849	−.003837	−.004667	−.005309	−.005770	4.40
4.50	−.001241	−.002098	−.002952	−.003707	−.004320	−.004783	4.50
4.60	−.000847	−.001532	−.002257	−.002929	−.003500	−.003952	4.60
4.70	−.000572	−.001109	−.001714	−.002303	−.002824	−.003254	4.70
4.80	−.000381	−.000797	−.001294	−.001802	−.002270	−.002671	4.80
4.90	−.000251	−.000567	−.000971	−.001403	−.001818	−.002185	4.90

t	SKEWNESS						t
	.6	.7	.8	.9	1.0	1.1	
.00	.529797	.589476	.636159	.668143	.683784	.681502	.00
.10	.551037	.584817	.605109	.611252	.602702	.579038	.10
.20	.549488	.560304	.558735	.544926	.519119	.481635	.20
.30	.529089	.520796	.502242	.474090	.437027	.391754	.30
.40	.493963	.470827	.440067	.402580	.359212	.310764	.40
.50	.448147	.414435	.375840	.333257	.287486	.239240	.50
.60	.395381	.355065	.312408	.268145	.222899	.177193	.60
.70	.338975	.295532	.251902	.208570	.165921	.124257	.70
.80	.281727	.238034	.195818	.155291	.116596	.079819	.80
.90	.225895	.184190	.145114	.108629	.074664	.043126	.90
1.00	.173207	.135106	.100308	.068570	.039660	.013358	1.00
1.10	.124896	.091451	.061567	.034861	.010097	-.010321	1.10
1.20	.081756	.053530	.028789	.007081	-.011981	-.028724	1.20
1.30	.044204	.021359	.001684	.015290	-.029950	-.042616	1.30
1.40	.012349	-.005260	-.020175	-.032830	-.043577	-.052703	1.40
1.50	-.013939	-.026683	-.037291	-.046132	-.053497	-.059624	1.50
1.60	-.034971	-.043367	-.050211	-.055782	-.060300	-.063943	1.60
1.70	-.051180	-.055831	-.059489	-.062338	-.064522	-.066158	1.70
1.80	-.063078	-.064616	-.065666	.066316	-.066641	-.066699	1.80
1.90	-.071211	-.070262	-.069251	-.068187	-.067079	-.065933	1.90
2.00	-.076134	-.073284	-.070716	-.068369	-.066198	-.064171	2.00
2.10	-.078386	-.074165	-.070483	-.067227	-.064311	-.061674	2.10
2.20	-.078470	-.073341	-.068926	-.065077	-.061682	-.058657	2.20
2.30	-.076847	-.071200	-.066369	-.062188	-.058529	-.055295	2.30
2.40	-.073926	-.068078	-.063089	-.058785	-.055032	-.051731	2.40
2.50	-.070062	-.064265	-.059318	-.055052	.051338	-.048078	2.50
2.60	-.065560	-.060001	-.055245	-.051139	-.047563	-.044424	2.60
2.70	-.060671	-.055484	-.051027	-.047167	-.043798	-.040837	2.70
2.80	-.055600	-.050872	-.046785	-.043228	-.040113	-.037366	2.80
2.90	-.050510	-.046291	-.042614	-.039394	-.036558	-.034049	2.90
3.00	-.045527	-.041835	-.038585	-.035715	-.033172	-.030910	3.00
3.10	-.040742	-.037573	-.034748	-.032229	-.029979	-.027964	3.10
3.20	-.036222	-.033553	-.031138	-.028959	-.026994	-.025220	3.20
3.30	-.032010	-.029807	-.027777	-.025918	-.024223	-.022679	3.30
3.40	-.028130	-.026350	-.024674	-.023113	-.021670	-.020340	3.40
3.50	-.024591	-.023190	-.021833	-.020542	-.019329	-.018198	3.50
3.60	-.021394	-.020323	-.019249	-.018200	-.017196	-.016243	3.60
3.70	-.018527	-.017741	-.016913	-.016078	-.015259	-.014468	3.70
3.80	-.015976	-.015430	-.014814	-.014165	-.013509	-.012861	3.80
3.90	-.013720	-.013373	-.012936	-.012447	-.011933	-.011411	3.90
4.00	-.011738	-.011552	-.011264	-.010911	-.010519	-.010107	4.00
4.10	-.010005	-.009948	-.009783	-.009543	-.009255	.008937	4.10
4.20	-.008499	-.008542	-.008475	-.008328	-.008127	-.007890	4.20
4.30	-.007196	-.007314	-.007324	-.007253	-.007125	-.006955	4.30
4.40	-.006073	-.006246	-.006315	-.006305	-.006236	-.006123	4.40
4.50	-.005110	-.005320	-.005433	-.005471	-.005449	-.005383	4.50
4.60	-.004288	-.004520	-.004665	-.004738	-.004754	-.004726	4.60
4.70	-.003588	-.003832	-.003998	-.004097	-.004142	-.004145	4.70
4.80	-.002994	-.003241	-.003419	-.003537	-.003604	-.003631	4.80
4.90	-.002492	-.002736	-.002919	-.003049	-.003132	-.003177	4.90

t	SKEWNESS						t
	.0	.1	.2	.3	.4	.5	
5.00	-.000164	-.000401	-.000724	-.001038	-.001450	-.001783	5.00
5.10	-.000105	-.000281	-.000537	-.000840	-.001153	-.001450	5.10
5.20	-.000067	-.000196	-.000396	-.000645	-.000914	-.001177	5.20
5.30	-.000042	-.000135	-.000291	-.000494	-.000722	-.000952	5.30
5.40	-.000026	-.000092	-.000212	-.000377	-.000568	-.000768	5.40
5.50	-.000016	-.000063	-.000154	-.000286	-.000446	-.000619	5.50
5.60	-.000010	-.000042	-.000111	-.000216	-.000349	-.000497	5.60
5.70	-.000006	-.000028	-.000080	-.000163	-.000272	-.000398	5.70
5.80	-.000004	-.000019	-.000057	-.000122	-.000212	-.000318	5.80
5.90	-.000002	-.000013	-.000041	-.000091	-.000164	-.000254	5.90
6.00	-.000001	-.000008	-.000029	-.000068	-.000127	-.000202	6.00
6.10		-.000005	-.000020	-.000051	-.000098	-.000160	6.10
6.20		-.000003	-.000014	-.000037	-.000075	-.000127	6.20
6.30		-.000002	-.000010	-.000028	-.000058	-.000101	6.30
6.40		-.000001	-.000007	-.000020	-.000044	-.000079	6.40
6.50		-.000001	-.000005	-.000015	-.000034	-.000063	6.50
6.60			-.000003	-.000011	-.000026	-.000049	6.60
6.70			-.000002	-.000008	-.000020	-.000039	6.70
6.80			-.000002	-.000006	-.000015	-.000030	6.80
6.90			-.000001	-.000004	-.000011	-.000024	6.90
7.00			-.000001	-.000003	-.000009	-.000018	7.00
7.10				-.000002	-.000006	-.000014	7.10
7.20				-.000002	-.000005	-.000011	7.20
7.30				-.000001	-.000004	-.000009	7.30
7.40				-.000001	-.000003	-.000007	7.40
7.50				-.000001	-.000002	-.000005	7.50
7.60					-.000002	-.000004	7.60
7.70					-.000001	-.000003	7.70
7.80					-.000001	-.000002	7.80
7.90					-.000001	-.000002	7.90
8.00						-.000001	8.00
8.10						-.000001	8.10
8.20						-.000001	8.20
8.30						-.000001	8.30
8.40							8.40
8.50							8.50
8.60							8.60
8.70							8.70
8.80							8.80
8.90							8.90
9.00							9.00
9.10							9.10
9.20							9.20
9.30							9.30
9.40							9.40
9.50							9.50
9.60							9.60
9.70							9.70
9.80							9.80
9.90							9.90

t	SKEWNESS						t
	.6	.7	.8	.9	1.0	1.1	
5.00	-.002069	-.002304	-.002488	-.002624	-.002719	-.002777	5.00
5.10	-.001714	-.001937	-.002117	-.002256	-.002357	-.002425	5.10
5.20	-.001417	-.001625	-.001798	-.001936	-.002041	-.002116	5.20
5.30	-.001169	-.001361	-.001526	-.001660	-.001765	-.001844	5.30
5.40	-.000962	-.001138	-.001292	-.001421	-.001525	-.001606	5.40
5.50	-.000790	-.000950	-.001093	-.001215	-.001317	-.001397	5.50
5.60	-.000647	-.000792	-.000923	-.001038	-.001135	-.001215	5.60
5.70	-.000530	-.000659	-.000779	-.000886	-.000978	-.001055	5.70
5.80	-.000432	-.000547	-.000656	-.000755	-.000842	-.000916	5.80
5.90	-.000353	-.000454	-.000552	-.000643	-.000724	-.000795	5.90
6.00	-.000287	-.000376	-.000464	-.000547	-.000622	-.000689	6.00
6.10	-.000233	-.000311	-.000389	-.000465	-.000534	-.000597	6.10
6.20	-.000189	-.000257	-.000326	-.000395	-.000459	-.000517	6.20
6.30	-.000153	-.000212	-.000273	-.000335	-.000393	-.000447	6.30
6.40	-.000124	-.000174	-.000229	-.000284	-.000337	-.000387	6.40
6.50	-.000100	-.000144	-.000191	-.000240	-.000288	-.000334	6.50
6.60	-.000080	-.000118	-.000160	-.000203	-.000247	-.000289	6.60
6.70	-.000065	-.000097	-.000133	-.000172	-.000211	-.000249	6.70
6.80	-.000052	-.000079	-.000111	-.000145	-.000180	-.000215	6.80
6.90	-.000042	-.000065	-.000092	-.000123	-.000154	-.000185	6.90
7.00	-.000033	-.000053	-.000077	-.000103	-.000131	-.000160	7.00
7.10	-.000027	-.000044	-.000064	-.000087	-.000112	-.000138	7.10
7.20	-.000021	-.000036	-.000053	-.000073	-.000095	-.000118	7.20
7.30	-.000017	-.000029	-.000044	-.000062	-.000081	-.000102	7.30
7.40	-.000014	-.000024	-.000037	-.000052	-.000069	-.000088	7.40
7.50	-.000011	-.000019	-.000030	-.000044	-.000059	-.000075	7.50
7.60	-.000009	-.000016	-.000025	-.000037	-.000050	-.000065	7.60
7.70	-.000007	-.000013	-.000021	-.000031	-.000043	-.000056	7.70
7.80	-.000005	-.000010	-.000017	-.000026	-.000036	-.000048	7.80
7.90	-.000004	-.000008	-.000014	-.000022	-.000031	-.000041	7.90
8.00	-.000003	-.000007	-.000012	-.000018	-.000026	-.000035	8.00
8.10	-.000003	-.000005	-.000010	-.000015	-.000022	-.000030	8.10
8.20	-.000002	-.000004	-.000008	-.000013	-.000019	-.000026	8.20
8.30	-.000002	-.000004	-.000007	-.000011	-.000016	-.000022	8.30
8.40	-.000001	-.000003	-.000005	-.000009	-.000014	-.000019	8.40
8.50	-.000001	-.000002	-.000004	-.000007	-.000011	-.000016	8.50
8.60	-.000001	-.000002	-.000004	-.000006	-.000010	-.000014	8.60
8.70	-.000001	-.000002	-.000003	-.000005	-.000008	-.000012	8.70
8.80	-.000001	-.000001	-.000002	-.000004	-.000007	-.000010	8.80
8.90		-.000001	-.000002	-.000004	-.000006	-.000009	8.90
9.00		-.000001	-.000002	-.000003	-.000005	-.000008	9.00
9.10		-.000001	-.000001	-.000003	-.000004	-.000006	9.10
9.20		-.000001	-.000001	-.000002	-.000004	-.000005	9.20
9.30			-.000001	-.000002	-.000003	-.000005	9.30
9.40			-.000001	-.000001	-.000003	-.000004	9.40
9.50			-.000001	-.000001	-.000002	-.000003	9.50
9.60			-.000001	-.000001	-.000002	-.000003	9.60
9.70				-.000001	-.000002	-.000002	9.70
9.80				-.000001	-.000001	-.000002	9.80
9.90				-.000001	-.000001	-.000002	9.90

t	SKEWNESS						t
	.0	.1	.2	.3	.4	.5	
10.00							10.00
10.10							10.10
10.20							10.20
10.30							10.30
10.40							10.40
10.50							10.50
10.60							10.60
10.70							10.70
10.80							10.80
10.90							10.90
11.00							11.00
11.10							11.10
11.20							11.20
11.30							11.30
11.40							11.40
11.50							11.50
11.60							11.60
11.70							11.70
11.80							11.80
11.90							11.90
12.00							12.00
12.10							12.10
12.20							12.20
12.30							12.30
12.40							12.40
12.50							12.50
12.60							12.60
12.70							12.70
12.80							12.80
12.90							12.90
13.00							13.00
13.10							13.10
13.20							13.20
13.30							13.30
13.40							13.40
13.50							13.50
13.60							13.60
13.70							13.70
13.80							13.80
13.90							13.90
14.00							14.00
14.10							14.10
14.20							14.20
14.30							14.30
14.40							14.40
14.50							14.50
14.60							14.60
14.70							14.70
14.80							14.80
14.90							14.90

	SKEWNESS						
t	.6	.7	.8	.9	1.0	1.1	t
10.00					-.000001	-.000002	10.00
10.10					-.000001	-.000001	10.10
10.20					-.000001	-.000001	10.20
10.30					-.000001	-.000001	10.30
10.40						-.000001	10.40
10.50						-.000001	10.50
10.60						-.000001	10.60
10.70						-.000001	10.70
10.80							10.80
10.90							10.90
11.00							11.00
11.10							11.10
11.20							11.20
11.30							11.30
11.40							11.40
11.50							11.50
11.60							11.60
11.70							11.70
11.80							11.80
11.90							11.90
12.00							12.00
12.10							12.10
12.20							12.20
12.30							12.30
12.40							12.40
12.50							12.50
12.60							12.60
12.70							12.70
12.80							12.80
12.90							12.90
13.00							13.00
13.10							13.10
13.20							13.20
13.30							13.30
13.40							13.40
13.50							13.50
13.60							13.60
13.70							13.70
13.80							13.80
13.90							13.90
14.00							14.00
14.10							14.10
14.20							14.20
14.30							14.30
14.40							14.40
14.50							14.50
14.60							14.60
14.70							14.70
14.80							14.80
14.90							14.90

t	SKEWNESS						t
	.0	.1	.2	.3	.4	.5	
-9.90							-9.90
-9.80							-9.80
-9.70							-9.70
-9.60							-9.60
-9.50							-9.50
-9.40							-9.40
-9.30							-9.30
-9.20							-9.20
-9.10							-9.10
-9.00							-9.00
-8.90							-8.90
-8.80							-8.80
-8.70							-8.70
-8.60							-8.60
-8.50							-8.50
-8.40							-8.40
-8.30							-8.30
-8.20							-8.20
-8.10							-8.10
-8.00							-8.00
-7.90							-7.90
-7.80							-7.80
-7.70							-7.70
-7.60							-7.60
-7.50							-7.50
-7.40							-7.40
-7.30							-7.30
-7.20							-7.20
-7.10							-7.10
-7.00							-7.00
-6.90							-6.90
-6.80							-6.80
-6.70							-6.70
-6.60							-6.60
-6.50							-6.50
-6.40							-6.40
-6.30							-6.30
-6.20							-6.20
-6.10							-6.10
-6.00	.000007						-6.00
-5.90	.000011						-5.90
-5.80	.000018						-5.80
-5.70	.000030						-5.70
-5.60	.000049						-5.60
-5.50	.000079	.000011					-5.50
-5.40	.000126	.000021					-5.40
-5.30	.000198	.000040					-5.30
-5.20	.000307	.000071					-5.20
-5.10	.000470	.000126					-5.10
-5.00	.000711	.000219	.000023				-5.00

t	SKEWNESS .6	.7	.8	.9	1.0	1.1	t
-9.90							-9.90
-9.80							-9.80
-9.70							-9.70
-9.60							-9.60
-9.50							-9.50
-9.40							-9.40
-9.30							-9.30
-9.20							-9.20
-9.10							-9.10
-9.00							-9.00
-8.90							-8.90
-8.80							-8.80
-8.70							-8.70
-8.60							-8.60
-8.50							-8.50
-8.40							-8.40
-8.30							-8.30
-8.20							-8.20
-8.10							-8.10
-8.00							-8.00
-7.90							-7.90
-7.80							-7.80
-7.70							-7.70
-7.60							-7.60
-7.50							-7.50
-7.40							-7.40
-7.30							-7.30
-7.20							-7.20
-7.10							-7.10
-7.00							-7.00
-6.90							-6.90
-6.80							-6.80
-6.70							-6.70
-6.60							-6.60
-6.50							-6.50
-6.40							-6.40
-6.30							-6.30
-6.20							-6.20
-6.10							-6.10
-6.00							-6.00
-5.90							-5.90
-5.80							-5.80
-5.70							-5.70
-5.60							-5.60
-5.50							-5.50
-5.40							-5.40
-5.30							-5.30
-5.20							-5.20
-5.10							-5.10
-5.00							-5.00

t	SKEWNESS .0	.1	.2	.3	.4	.5	t
-4.90	.001062	.000373	.000050				-4.90
-4.80	.001567	.000621	.000106				-4.80
-4.70	.002283	.001014	.000216				-4.70
-4.60	.003283	.001623	.000424				-4.60
-4.50	.004660	.002547	.000801				-4.50
-4.40	.006526	.003919	.001461	.000165			-4.40
-4.30	.009015	.005910	.002576	.000414			-4.30
-4.20	.012280	.008735	.004392	.000965			-4.20
-4.10	.016488	.012652	.007249	.002098			-4.10
-4.00	.021814	.017953	.011585	.004275	.000270		-4.00
-3.90	.028424	.024952	.017935	.008198	.000964		-3.90
-3.80	.036456	.033952	.026904	.014839	.002892		-3.80
-3.70	.045994	.045210	.039109	.025429	.007498		-3.70
-3.60	.057030	.058872	.055089	.041352	.017134		-3.60
-3.50	.069433	.074908	.075174	.063929	.035043	.003363	-3.50
-3.40	.082896	.093028	.099316	.094085	.064923	.014067	-3.40
-3.30	.096898	.112599	.126916	.131914	.109964	.042678	-3.30
-3.20	.110664	.132568	.156637	.176212	.171459	.101860	-3.20
-3.10	.123133	.151405	.186283	.224065	.247286	.201253	-3.10
-3.00	.132955	.167089	.212736	.270631	.330741	.340245	-3.00
-2.90	.138504	.177145	.232029	.309206	.410203	.502693	-2.90
-2.80	.137931	.178748	.239565	.331680	.470001	.656676	-2.80
-2.70	.129262	.168916	.230499	.329387	.492520	.760190	-2.70
-2.60	.110533	.144769	.200274	.294297	.468964	.771242	-2.60
-2.50	.079973	.103864	.145268	.220393	.364364	.659133	-2.50
-2.40	.036225	.044574	.063483	.105036	.197539	.413477	-2.40
-2.30	-.021415	-.033521	-.044820	-.049946	-.033760	.048415	-2.30
-2.20	-.092745	-.129292	-.176631	-.237650	-.313972	-.398860	-2.20
-2.10	-.176458	-.239792	-.325968	-.446158	-.618703	-.874547	-2.10
-2.00	-.269955	-.360149	-.483969	-.659320	-.917573	-1.317793	-2.00
-1.90	-.369279	-.483634	-.639366	-.858210	-1.178217	-1.671021	-1.90
-1.80	-.469154	-.601945	-.779314	-1.023098	-1.370447	-1.888901	-1.80
-1.70	-.563157	-.705722	-.890510	-1.135663	-1.470408	-1.944452	-1.70
-1.60	-.644051	-.785244	-.960510	-1.181176	-1.463555	-1.831597	-1.60
-1.50	-.704252	-.831283	-.979070	-1.150310	-1.346367	-1.564257	-1.50
-1.40	-.736420	-.836023	-.939371	-1.040370	-1.126539	-1.172695	-1.40
-1.30	-.734126	-.793947	-.838954	-.855748	-.821731	-.698106	-1.30
-1.20	-.692545	-.702593	-.680247	-.607581	-.457192	-.186561	-1.20
-1.10	-.609093	-.563089	-.451878	-.312649	-.062669	.316737	-1.10
-1.00	-.483941	-.380366	-.221798	.008309	.330924	.772172	-1.00
- .90	-.320340	-.163025	.050847	.332640	.694644	1.149001	- .90
- .80	-.124683	.077163	.329834	.637935	1.004102	1.427081	- .80
- .70	.093707	.326094	.596972	.904033	1.241282	1.597145	- .70
- .60	.323095	.568419	.834958	1.114676	1.395475	1.659900	- .60
- .50	.550102	.788781	1.028829	1.258656	1.463382	1.624302	- .50
- .40	.760699	.973094	1.167154	1.330391	1.448486	1.505356	- .40
- .30	.941303	1.109698	1.208438	1.329919	1.359877	1.321796	- .30
- .20	1.079904	1.190311	1.253522	1.262385	1.210756	1.093886	- .20
- .10	1.167080	1.210653	1.201474	1.137122	1.016802	.841546	- .10
.00	1.196827	1.170712	1.093148	.966479	.794593	.582903	.00

t	SKEWNESS						t
	.6	.7	.8	.9	1.0	1.1	
-4.90							-4.90
-4.80							-4.80
-4.70							-4.70
-4.60							-4.60
-4.50							-4.50
-4.40							-4.40
-4.30							-4.30
-4.20							-4.20
-4.10							-4.10
-4.00							-4.00
-3.90							-3.90
-3.80							-3.80
-3.70							-3.70
-3.60							-3.60
-3.50							-3.50
-3.40							-3.40
-3.30							-3.30
-3.20	.020824						-3.20
-3.10	.028526						-3.10
-3.00	.147081						-3.00
-2.90	.395781						-2.90
-2.80	.759352						-2.80
-2.70	1.124906	.812211					-2.70
-2.60	1.347521	2.017497					-2.60
-2.50	1.309026	2.763730					-2.50
-2.40	.962273	2.549975	7.177420				-2.40
-2.30	.343284	1.433681	6.605834				-2.30
-2.20	-.445103	-.193679	2.155290	47.637462			-2.20
-2.10	-1.265630	-1.862554	-2.531723	3.046070			-2.10
-2.00	-1.981650	-3.194250	-5.763751	-12.89341			-2.00
-1.90	-2.485037	-3.971531	-7.147354	-16.21020	-75.41929		-1.90
-1.80	-2.712264	-4.138399	-6.952061	-13.86402	-40.95566	-1703.342	-1.80
-1.70	-2.648307	-3.762105	-5.690392	-9.514587	-19.03718	-55.10019	-1.70
-1.60	-2.320871	-2.983216	-3.883787	-5.027707	-5.674726	4.547146	-1.60
-1.50	-1.788663	-1.969634	-1.954736	-1.253452	1.962024	16.717548	-1.50
-1.40	-1.127406	-.882058	-.194341	1.521847	5.860034	17.978712	-1.40
-1.30	-.416546	.147390	1.230284	3.306926	7.396594	15.872761	-1.30
-1.20	.271442	1.024888	2.251319	4.247538	7.511627	12.840068	-1.20
-1.10	.878276	1.695880	2.871243	4.536417	6.834117	9.805846	-1.10
-1.00	1.363543	2.140438	3.136250	4.364950	5.774305	7.121962	-1.00
- .90	1.705324	2.365735	3.114882	3.899959	4.590501	4.896272	- .90
- .80	1.898524	2.397668	2.882507	3.275111	3.437122	3.128725	- .80
- .70	1.951749	2.273015	2.511075	2.590410	2.398938	1.773397	- .70
- .60	1.883508	2.032867	2.063195	1.915738	1.515225	.768222	- .60
- .50	1.718347	1.717661	1.589403	1.296049	.796593	.049368	- .50
- .40	1.483373	1.363810	1.127644	.756864	.236529	-.442085	- .40
- .30	1.205400	1.001743	.704131	.309331	-.180839	-.757437	- .30
- .20	.908852	.655086	.334973	-.045445	-.475651	-.939707	- .20
- .10	.614408	.340679	.028099	-.312925	-.669095	-1.023989	- .10
.00	.338310	.069165	-.214782	-.502446	-.781467	-1.038291	.00

	SKEWNESS						
t	.0	.1	.2	.3	.4	.5	t
.00	1.196827	1.170712	1.093148	.966479	.794593	.582903	.00
.10	1.167080	1.074614	.938367	.764534	.560223	.333295	.10
.20	1.079904	.930144	.749299	.545828	.328197	.104721	.20
.30	.941303	.747971	.539336	.324239	.110656	-.094324	.30
.40	.760699	.540662	.321978	.112068	-.083069	-.258707	.40
.50	.550102	.321613	.109835	-.080603	-.246603	-.386303	.50
.60	.323095	.103981	-.086178	-.246284	-.376418	-.477487	.60
.70	.093707	-.100271	-.257417	-.380193	-.471536	-.534582	.70
.80	-.124683	-.281142	-.397824	-.480139	-.533110	-.561286	.80
.90	-.320340	-.431125	-.503997	-.546236	-.563934	-.562147	.90
1.00	-.483941	-.545473	-.575047	-.580507	-.567948	-.542106	1.00
1.10	-.609093	-.622220	-.612288	-.586411	-.549764	-.506123	1.10
1.20	-.692545	-.661971	-.618802	-.568366	-.514257	-.458890	1.20
1.30	-.734126	-.667522	-.598957	-.531300	-.466219	-.404633	1.30
1.40	-.736420	-.643356	-.557907	-.480250	-.410109	-.346993	1.40
1.50	-.704252	-.595087	-.501121	-.420037	-.349867	-.288967	1.50
1.60	-.644051	-.528901	-.433987	-.355027	-.288813	-.232901	1.60
1.70	-.563157	-.451048	-.361482	-.288969	-.229601	-.180529	1.70
1.80	-.469154	-.367406	-.287948	-.224909	-.174230	-.133032	1.80
1.90	-.369279	-.283159	-.216948	-.165165	-.124086	-.091107	1.90
2.00	-.269955	-.202581	-.151207	-.111359	-.080012	-.055060	2.00
2.10	-.176458	-.128927	-.095701	-.064481	-.042394	-.024879	2.10
2.20	-.092745	-.064425	-.042334	-.024976	-.011244	-.000317	2.20
2.30	-.021415	-.010335	-.000806	.007156	.013705	.019042	2.30
2.40	.036225	.032924	.032034	.032252	.032923	.033717	2.40
2.50	.079973	.065624	.056680	.050897	.047011	.044288	2.50
2.60	.110533	.088555	.073917	.063826	.056636	.051348	2.60
2.70	.129262	.102860	.084711	.071857	.062491	.055483	2.70
2.80	.137931	.109882	.090113	.075826	.065250	.057241	2.80
2.90	.138504	.111035	.091177	.076544	.065547	.057123	2.90
3.00	.132955	.107697	.088906	.074760	.063956	.055574	3.00
3.10	.123133	.101141	.084210	.071145	.060979	.052981	3.10
3.20	.110664	.092480	.077881	.066276	.057048	.049668	3.20
3.30	.096898	.082650	.070581	.060636	.052522	.045906	3.30
3.40	.082896	.072394	.062845	.054615	.047688	.041910	3.40
3.50	.069433	.062277	.055085	.048518	.042774	.037852	3.50
3.60	.057030	.052701	.047604	.042570	.037951	.033860	3.60
3.70	.045994	.043925	.040610	.036935	.033342	.030027	3.70
3.80	.036456	.036097	.034233	.031718	.029031	.026420	3.80
3.90	.028424	.029271	.028537	.026980	.025070	.023080	3.90
4.00	.021814	.023440	.023543	.022747	.021485	.020029	4.00
4.10	.016488	.018546	.019233	.019020	.018283	.017274	4.10
4.20	.012280	.014507	.015566	.015781	.015455	.014813	4.20
4.30	.009015	.011224	.012487	.012997	.012982	.012634	4.30
4.40	.006526	.008592	.009933	.010630	.010842	.010722	4.40
4.50	.004660	.006510	.007837	.008636	.009004	.009056	4.50
4.60	.003283	.004884	.006136	.006972	.007438	.007615	4.60
4.70	.002283	.003629	.004768	.005594	.006113	.006375	4.70
4.80	.001567	.002671	.003678	.004463	.005000	.005316	4.80
4.90	.001062	.001948	.002818	.003541	.004071	.004416	4.90

t	SKEWNESS						t
	.6	.7	.8	.9	1.0	1.1	
.00	.338310	.069165	-.214782	-.502446	-.781467	-1.038291	.00
.10	.092212	-.154102	-.396402	-.625246	-.831115	-1.004521	.10
.20	-.116564	-.328001	-.522394	-.693075	-.833942	-.939499	.20
.30	-.284508	-.454578	-.600057	-.717270	-.803289	-.855884	.30
.40	-.411246	-.538066	-.637395	-.708191	-.750026	-.762993	.40
.50	-.498817	-.584031	-.642421	-.674916	-.682779	-.667508	.50
.60	-.550963	-.598678	.622683	-.625131	-.608205	-.574058	.60
.70	-.492063	-.588315	.584965	-.565132	-.531294	-.485703	.70
.80	-.568715	-.558962	.535134	-.499924	-.455656	-.404319	.80
.90	-.545052	-.516090	.478086	-.433353	-.383781	-.330911	.90
1.00	-.506662	-.464476	-.417768	-.368259	-.317280	-.265855	1.00
1.10	-.458242	-.408129	-.357245	-.306646	-.257083	-.251529	1.10
1.20	-.403880	-.350288	-.298798	-.249826	-.203605	-.160241	1.20
1.30	-.346995	-.293466	-.244031	-.198567	-.156889	-.118778	1.30
1.40	-.290326	-.239514	-.193988	-.153216	-.116712	-.084036	1.40
1.50	-.235962	-.189710	-.149257	-.113803	-.082677	-.055313	1.50
1.60	-.185405	-.144850	-.110070	-.080131	-.054280	-.031903	1.60
1.70	-.139641	-.105337	-.076391	-.051849	-.030957	-.013117	1.70
1.80	-.099226	-.071267	-.047991	-.028508	-.012128	.001690	1.80
1.90	-.064367	-.042504	-.024507	.009608	.002780	.013113	1.90
2.00	-.035003	-.018750	-.005495	.005372	.014313	.021689	2.00
2.10	-.010873	.000404	.009531	.016944	.022980	.027897	2.10
2.20	.008421	.015432	.021068	.025602	.029244	.032158	2.20
2.30	.023361	.026834	.029607	.031802	.033518	.034838	2.30
2.40	.034472	.035112	.035610	.035960	.036170	.036251	2.40
2.50	.042290	.040751	.039506	.038450	.037517	.036664	2.50
2.60	.047337	.044203	.041681	.039598	.037833	.036305	2.60
2.70	.050105	.045878	.042480	.039690	.037353	.035361	2.70
2.80	.051042	.046145	.042202	.038969	.036272	.033988	2.80
2.90	.050547	.045324	.041104	.037639	.034753	.032315	2.90
3.00	.048971	.043689	.039402	.035873	.032930	.030444	3.00
3.10	.046611	.041474	.037279	.033811	.030911	.028459	3.10
3.20	.043716	.038869	.034881	.031567	.028784	.026424	3.20
3.30	.040491	.036031	.032329	.029231	.026616	.024391	3.30
3.40	.037099	.033084	.029716	.026875	.024461	.022396	3.40
3.50	.033670	.030124	.027115	.024552	.022358	.020470	3.50
3.60	.030299	.027226	.024581	.022303	.020336	.018631	3.60
3.70	.027059	.024443	.022154	.020157	.018416	.016894	3.70
3.80	.024000	.021812	.019861	.018134	.016610	.015266	3.80
3.90	.021152	.019356	.017719	.016245	.014927	.013752	3.90
4.00	.018534	.017090	.015738	.014497	.013370	.012353	4.00
4.10	.016154	.015018	.013921	.001289	.011938	.011067	4.10
4.20	.014009	.013141	.012268	.011425	.010629	.009890	4.20
4.30	.012093	.011452	.010773	.010095	.009439	.008819	3.30
4.40	.010393	.009942	.009429	.008894	.008361	.007847	4.40
4.50	.008896	.008600	.008227	.007815	.007390	.006968	4.50
4.60	.007585	.007415	.007158	.006850	.006517	.006176	4.60
4.70	.006443	.006373	.006211	.005990	.005736	.005465	4.70
4.80	.005455	.005462	.005375	.005226	.005038	.004828	4.80
4.90	.004603	.004667	.004641	.004551	.004418	.004258	4.90

	SKEWNESS						
t	.0	.1	.2	.3	.4	.5	t
5.00	.000711	.001408	.002144	.002794	.003300	.003654	5.00
5.10	.000470	.001009	.001621	.002193	.002664	.003014	5.10
5.20	.000307	.000717	.001217	.001713	.002142	.002477	5.20
5.30	.000198	.000505	.000909	.001331	.001715	.002029	5.30
5.40	.000126	.000353	.000674	.001030	.001368	.001657	5.40
5.50	.000079	.000245	.000497	.000793	.001087	.001349	5.50
5.60	.000049	.000168	.000365	.000608	.000861	.001095	5.60
5.70	.000030	.000115	.000266	.000464	.000680	.000887	5.70
5.80	.000018	.000078	.000193	.000352	.000535	.000716	5.80
5.90	.000011	.000052	.000139	.000267	.000419	.000577	5.90
6.00	.000007	.000035	.000100	.000202	.000328	.000463	6.00
6.10		.000023	.000071	.000151	.000256	.000371	6.10
6.20		.000015	.000051	.000113	.000199	.000297	6.20
6.30		.000010	.000036	.000084	.000154	.000237	6.30
6.40		.000006	.000025	.000063	.000119	.000188	6.40
6.50		.000004	.000018	.000046	.000092	.000150	6.50
6.60			.000012	.000034	.000070	.000119	6.60
6.70			.000009	.000025	.000054	.000094	6.70
6.80			.000006	.000018	.000041	.000074	6.80
6.90			.000004	.000014	.000032	.000058	6.90
7.00			.000003	.000010	.000024	.000046	7.00
7.10			.000002	.000007	.000018	.000036	7.10
7.20			.000001	.000005	.000014	.000028	7.20
7.30			.000001	.000004	.000010	.000022	7.30
7.40			.000001	.000003	.000008	.000017	7.40
7.50				.000002	.000006	.000013	7.50
7.60				.000001	.000004	.000010	7.60
7.70				.000001	.000003	.000008	7.70
7.80				.000001	.000003	.000006	7.80
7.90					.000002	.000005	7.90
8.00					.000001	.000004	8.00
8.10					.000001	.000003	8.10
8.20					.000001	.000002	8.20
8.30					.000001	.000002	8.30
8.40						.000001	8.40
8.50						.000001	8.50
8.60						.000001	8.60
8.70						.000001	8.70
8.80							8.80
8.90							8.90
9.00							9.00
9.10							9.10
9.20							9.20
9.30							9.30
9.40							9.40
9.50							9.50
9.60							9.60
9.70							9.70
9.80							9.80
9.90							9.90

t	SKEWNESS						t
	.6	.7	.8	.9	1.0	1.1	
5.00	.003872	.003977	.003997	.003954	.003867	.003750	5.00
5.10	.003247	.003381	.003436	.003430	.003380	.003299	5.10
5.20	.002715	.002867	.002947	.002970	.002949	.002897	5.20
5.30	.002265	.002426	.002522	.002567	.002570	.002542	5.30
5.40	.001884	.002048	.002155	.002215	.002236	.002227	5.40
5.50	.001563	.001725	.001838	.001908	.001943	.001950	5.50
5.60	.001294	.001450	.001565	.001642	.001686	.001705	5.60
5.70	.001068	.001217	.001330	.001410	.001462	.001489	5.70
5.80	.000880	.001019	.001129	.001210	.001266	.001299	5.80
5.90	.000724	.000852	.000956	.001037	.001095	.001133	5.90
6.00	.000594	.000711	.000809	.000887	.000946	.000987	6.00
6.10	.000486	.000592	.000684	.000758	.000816	.000859	6.10
6.20	.000397	.000493	.000577	.000648	.000704	.000747	6.20
6.30	.000324	.000409	.000486	.000552	.000606	.000649	6.30
6.40	.000264	.000339	.000409	.000470	.000522	.000563	6.40
6.50	.000215	.000281	.000344	.000400	.000449	.000489	6.50
6.60	000174	.000232	.000289	.000340	.000386	.000424	6.60
6.70	.000141	.000192	.000242	.000289	.000331	.000367	6.70
6.80	.000114	.000158	.000203	.000245	.000284	.000318	6.80
6.90	.000092	.000130	.000170	.000208	.000244	.000275	6.90
7.00	.000074	.000107	.000142	000176	.000209	.000238	7.00
7.10	.000060	.000088	000118	.000149	.000179	.000205	7.10
7.20	.000048	.000072	.000099	.000126	.000153	.000177	7.20
7.30	.000039	.000059	.000082	.000107	.000131	.000153	7.30
7.40	.000031	.000048	.000069	.000090	.000112	.000132	7.40
7.50	.000025	.000040	.000057	.000076	.000095	.000114	7.50
7.60	.000020	.000032	.000047	.000064	.000081	.000098	7.60
7.70	.000016	.000026	.000039	.000054	.000069	.000085	7.70
7.80	.000013	.000022	.000033	.000045	.000059	.000073	7.80
7.90	.000010	.000018	.000027	.000038	.000050	.000063	7.90
8.00	.000008	.000014	.000022	.000032	.000043	.000054	8.00
8.10	.000006	.000012	.000019	.000027	.000036	.000046	8.10
8.20	.000005	.000009	.000015	.000023	.000031	.000040	8.20
8.30	.000004	.000008	.000013	.000019	.000026	.000034	8.30
8.40	.000003	.000006	.000011	.000016	.000022	.000029	8.40
8.50	.000002	.000005	.000009	.000013	.000019	.000025	8.50
8.60	.000002	.000004	.000007	.000011	.000016	.000022	8.60
8.70	.000002	.000003	.000006	.000009	.000014	.000019	8.70
8.80	.000001	.000003	.000005	.000008	.000912	.000016	8.80
8.90	.000001	.000002	.000004	.000007	.000010	.000014	8.90
9.00	.000001	.000002	.000003	.000006	.000008	.000012	9.00
9.10	.000001	.000001	.000003	.000005	.000007	.000010	9.10
9.20		.000001	.000002	.000004	.000006	.000009	9.20
9.30		.000001	.000002	.000003	.000005	.000007	9.30
9.40		.000001	.000002	.000003	.000004	.000006	9.40
9.50		.000001	.000001	.000002	.000004	.000005	9.50
9.60			.000001	.000002	.000003	.000005	9.60
9.70			.000001	.000002	.000003	.000004	9.70
9.80			.000001	.000001	.000002	.000003	9.80
9.90			.000001	.000001	.000002	.000003	9.90

t	SKEWNESS						t
	.6	.7	.8	.9	1.0	1.1	
10.00							10.00
10.10							10.10
10.20							10.20
10.30							10.30
10.40							10.40
10.50							10.50
10.60							10.60
10.70							10.70
10.80							10.80
10.90							10.90
11.00							11.00
11.10							11.10
11.20							11.20
11.30							11.30
11.40							11.40
11.50							11.50
11.60							11.60
11.70							11.70
11.80							11.80
11.90							11.90
12.00							12.00
12.10							12.10
12.20							12.20
12.30							12.30
12.40							12.40
12.50							12.50
12.60							12.60
12.70							12.70
12.80							12.80
12.90							12.90
13.00							13.00
13.10							13.10
13.20							13.20
13.30							13.30
13.40							13.40
13.50							13.50
13.60							13.60
13.70							13.70
13.80							13.80
13.90							13.90
14.00							14.00
14.10							14.10
14.20							14.20
14.30							14.30
14.40							14.40
14.50							14.50
14.60							14.60
14.70							14.70
14.80							14.80
14.90							14.90

	SKEWNESS						
t	.6	.7	.8	.9	1.0	1.1	t
10.00				.000001	.000002	.000002	10.00
10.10				.000001	.000001	.000002	10.10
10.20				.000001	.000001	.000002	10.20
10.30				.000001	.000001	.000002	10.30
10.40					.000001	.000001	10.40
10.50					.000001	.000001	10.50
10.60					.000001	.000001	10.60
10.70						.000001	10.70
10.80						.000001	10.80
10.90						.000001	10.90
11.00						.000001	11.00
11.10							11.10
11.20							11.20
11.30							11.30
11.40							11.40
11.50							11.50
11.60							11.60
11.70							11.70
11.80							11.80
11.90							11.90
12.00							12.00
12.10							12.10
12.20							12.20
12.30							12.30
12.40							12.40
12.50							12.50
12.60							12.60
12.70							12.70
12.80							12.80
12.90							12.90
13.00							13.00
13.10							13.10
13.20							13.20
13.30							13.30
13.40							13.40
13.50							13.50
13.60							13.60
13.70							13.70
13.80							13.80
13.90							13.90
14.00							14.00
14.10							14.10
14.20							14.20
14.30							14.30
14.40							14.40
14.50							14.50
14.60							14.60
14.70							14.70
14.80							14.80
14.90							14.90

t	SKEWNESS .0	.1	.2	.3	.4	.5	t
-9.90							-9.90
-9.80							-9.80
-9.70							-9.70
-9.60							-9.60
-9.50							-9.50
-9.40							-9.40
-9.30							-9.30
-9.20							-9.20
-9.10							-9.10
-9.00							-9.00
-8.90							-8.90
-8.80							-8.80
-8.70							-8.70
-8.60							-8.60
-8.50							-8.50
-8.40							-8.40
-8.30							-8.30
-8.20							-8.20
-8.10							-8.10
-8.00							-8.00
-7.90							-7.90
-7.80							-7.80
-7.70							-7.70
-7.60							-7.60
-7.50							-7.50
-7.40							-7.40
-7.30							-7.30
-7.20							-7.20
-7.10							-7.10
-7.00							-7.00
-6.90							-6.90
-6.80							-6.80
-6.70							-6.70
-6.60							-6.60
-6.50							-6.50
-6.40							-6.40
-6.30							-6.30
-6.20							-6.20
-6.10							-6.10
-6.00	.000035						-6.00
-5.90	.000057						-5.90
-5.80	.000093						-5.80
-5.70	.000149						-5.70
-5.60	.000237						-5.60
-5.50	.000372	.000073					-5.50
-5.40	.000575	.000134					-5.40
-5.30	.000879	.000238					-5.30
-5.20	.001326	.000415					-5.20
-5.10	.001974	.000709					-5.10
-5.00	.002899	.001185	.000185				-5.00

	SKEWNESS						
t	.6	.7	.8	.9	1.0	1.1	t
-9.90							-9.90
-9.80							-9.80
-9.70							-9.70
-9.60							-9.60
-9.50							-9.50
-9.40							-9.40
-9.30							-9.30
-9.20							-9.20
-9.10							-9.10
-9.00							-9.00
-8.90							-8.90
-8.80							-8.80
-8.70							-8.70
-8.60							-8.60
-8.50							-8.50
-8.40							-8.40
-8.30							-8.30
-8.20							-8.20
-8.10							-8.10
-8.00							-8.00
-7.90							-7.90
-7.80							-7.80
-7.70							-7.70
-7.60							-7.60
-7.50							-7.50
-7.40							-7.40
-7.30							-7.30
-7.20							-7.20
-7.10							-7.10
-7.00							-7.00
-6.90							-6.90
-6.80							-6.80
-6.70							-6.70
-6.60							-6.60
-6.50							-6.50
-6.40							-6.40
-6.30							-6.30
-6.20							-6.20
-6.10							-6.10
-6.00							-6.00
-5.90							-5.90
-5.80							-5.80
-5.70							-5.70
-5.60							-5.60
-5.50							-5.50
-5.40							-5.40
-5.30							-5.30
-5.20							-5.20
-5.10							-5.10
-5.00							-5.00

	SKEWNESS						
t	.0	.1	.2	.3	.4	.5	t
−4.90	.004199	.001939	.000387				−4.90
−4.80	.005998	.003107	.000775				−4.80
−4.70	.008445	.004872	.001494				−4.70
−4.60	.011715	.007477	.002771				−4.60
−4.50	.016008	.011228	.004953				−4.50
−4.40	.021533	.016492	.008532	.001582			−4.40
−4.30	.028497	.023686	.014175	.003653			−4.30
−4.20	.037077	.033246	.022723	.007821			−4.20
−4.10	.047385	.045577	.035150	.015601			−4.10
−4.00	.059421	.060973	.052467	.029110	.003698		−4.00
−3.90	.073016	.079514	.075548	.050964	.011373		−3.90
−3.80	.087768	.100934	.104865	.083927	.029557		−3.80
−3.70	.102971	.124473	.140154	.130216	.066481		−3.70
−3.60	.117547	.148727	.180029	.190495	.131709		−3.60
−3.50	.130002	.171523	.221615	.262656	.232800	.054837	−3.50
−3.40	.138396	.189842	.260269	.340701	.370353	.176319	−3.40
−3.30	.140362	.199841	.289515	.413951	.532885	.418003	−3.30
−3.20	.133185	.196991	.301268	.467184	.693686	.782995	−3.20
−3.10	.113952	.176376	.286440	.481803	.811755	1.204809	−3.10
−3.00	.079773	.133159	.235965	.438269	.837828	1.548187	−3.00
−2.90	.028105	.063208	.142169	.319573	.724933	1.646581	−2.90
−2.80	−.042873	−.036154	.000387	.115261	.441138	1.361310	−2.80
−2.70	−.133814	−.165339	−.189409	−.174737	−.018815	.637016	−2.70
−2.60	−.243764	−.321742	−.421241	−.537330	−.629273	−.468718	−2.60
−2.50	−.369738	−.499140	−.682317	−.945332	−1.319247	−1.791490	−2.50
−2.40	−.506424	−.687411	−.953041	−1.358873	−2.008265	−3.097791	−2.40
−2.30	−.646042	−.872660	−1.208037	−1.729252	−2.591569	−4.140341	−2.30
−2.20	−.778443	−1.037858	−1.418202	−2.004833	−2.971517	−4.713838	−2.20
−2.10	−.891503	−1.163872	−1.553678	−2.138166	−3.071825	−4.696711	−2.10
−2.00	−.971838	−1.231866	−1.587421	−2.093193	−2.851085	−4.070717	−2.00
−1.90	−1.005825	−1.223902	−1.498933	−1.851350	−2.310517	−2.917258	−1.90
−1.80	−.980903	−1.126606	−1.277630	−1.415493	−1.493788	−1.394949	−1.80
−1.70	−.887018	−.932582	−.925326	−.810963	−.480907	.293881	−1.70
−1.60	−.718128	−.642320	−.457386	−.083541	.623876	1.939386	−1.60
−1.50	−.473549	−.265293	.097713	.705429	1.706836	3.356043	−1.50
−1.40	−.158975	.179815	.700455	1.486002	2.659884	4.405508	−1.40
−1.30	.212999	.665911	1.303840	2.187685	3.393671	5.008519	−1.30
−1.20	.623011	1.159322	1.857979	2.747403	3.846966	5.146248	−1.20
−1.10	1.045802	1.622521	2.220861	3.116326	3.991435	4.853130	−1.10
−1.00	1.451824	2.017519	2.635041	3.264706	3.831751	4.204047	−1.00
− .90	1.809510	2.309533	2.788437	3.184231	3.401691	3.298929	− .90
− .80	2.088004	2.470542	2.760718	2.887791	2.757297	2.247383	− .80
− .70	2.260116	2.482291	2.552957	2.406880	1.968436	1.155361	− .70
− .60	2.305168	2.338384	2.181623	1.787195	1.110037	.115043	− .60
− .50	2.211410	2.045193	1.676618	1.083101	.254115	−.801603	− .50
− .40	1.977699	1.621472	1.077996	.351751	−.536640	−1.545714	− .40
− .30	1.614197	1.096704	.438916	−.352453	−1.213257	−2.091401	− .30
− .20	1.141970	.508372	−.214335	−.982478	−1.742756	−2.433363	− .20
− .10	.591463	−.101530	−.815746	−1.501985	−2.108462	−2.583054	− .10
.00	.000000	−.690340	−1.334227	−1.887190	−2.308710	−2.564153	.00

t	SKEWNESS .6	.7	.8	.9	1.0	1.1	t
-4.90							-4.90
-4.80							-4.80
-4.70							-4.70
-4.60							-4.60
-4.50							-4.50
-4.40							-4.40
-4.30							-4.30
-4.20							-4.20
-4.10							-4.10
-4.00							-4.00
-3.90							-3.90
-3.80							-3.80
-3.70							-3.70
-3.60							-3.60
-3.50							-3.50
-3.40							-3.40
-3.30							-3.30
-3.20	.842257						-3.20
-3.10	.588693						-3.10
-3.00	1.849903						-3.00
-2.90	3.201920						-2.90
-2.80	3.878637						-2.80
-2.70	3.179643	11.313829					-2.70
-2.60	1.068006	11.097192					-2.60
-2.50	-1.918392	2.984160					-2.50
-2.40	-4.953572	-7.118615	30.500628				-2.40
-2.30	-7.249070	-14.50980	-32.96668				-2.30
-2.20	-8.283837	-17.22664	-49.96355	-761.6006			-2.20
-2.10	7.894204	-15.50871	-41.08784	-260.2257			-2.10
-2.00	-6.243303	-10.76401	-23.00499	-80.70170			-2.00
-1.90	-3.717972	-4.695233	5.185098	3.299597	424.36017		-1.90
-1.80	.802582	1.237087	8.199275	27.923815	274.10548	151298.68	-1.80
-1.70	2.033508	6.046683	16.148280	46.253906	170.84286	1305.4169	-1.70
-1.60	4.414962	9.245487	19.275092	42.139264	101.07146	238.57941	-1.60
-1.50	6.100725	10.754799	18.822519	32.911782	54.936664	46.225475	-1.50
-1.40	6.990201	10.774953	16.111270	22.634124	25.290674	-10.47424	-1.40
-1.30	7.105003	9.657556	12.273179	13.324788	6.986257	-28.00302	-1.30
-1.20	6.557023	7.802318	8.156143	5.816354	-3.652442	-31.23638	-1.20
-1.10	5.512210	5.585612	4.322899	.283832	-9.225001	-28.90925	-1.10
-1.00	4.156897	3.318942	1.094040	-3.433856	-11.54861	-24.60908	-1.00
- .90	2.670779	1.231482	-1.394055	-5.643201	-11.87573	-19.91430	- .90
- .80	1.208217	- .530420	-3.132533	-6.687731	-11.05453	-15.51779	- .80
- .70	- .112213	-1.893330	-4.191323	-6.890713	-9.646454	-11.69445	- .70
- .60	-1.211125	-2.841685	-4.681987	-6.528740	-8.012009	-8.516467	- .60
- .50	-2.046433	-3.401428	-4.730771	-5.823350	-6.372745	-5.959217	- .50
- .40	-2.608007	-3.624745	-4.461008	-4.952497	-4.855811	-3.955431	- .40
- .30	-2.910524	-3.577221	-3.982722	-4.006892	-3.525641	-2.423242	- .30
- .20	-2.985855	-3.327957	-3.387561	-3.098185	-2.406236	-1.280497	- .20
- .10	-2.875929	-2.942594	-2.747366	-2.267305	-1.496634	- .451666	- .10
.00	-2.626655	-2.478942	-2.115071	-1.542070	- .781467	.129293	.00

t	SKEWNESS						t
	.0	.1	.2	.3	.4	.5	
.00	.000000	-.690340	-1.334227	-1.887190	-2.308710	-2.564153	.00
.10	-.591463	-1.218512	-1.740924	-2.127279	-2.354389	-2.407989	.10
.20	-1.141970	-1.652927	-2.017921	-2.223557	-2.265822	-2.149357	.20
.30	-1.614197	-1.969413	-2.158675	-2.187627	-2.069421	-1.823040	.30
.40	-1.977699	-2.154258	-2.167343	-2.038926	-1.794478	-1.461172	.40
.50	-2.211410	-2.204630	-2.057225	-1.801983	-1.470374	-1.091458	.50
.60	-2.305168	-2.127944	-1.848575	-1.503696	-1.124365	- .736196	.60
.70	-2.260116	-1.940344	-1.566096	-1.170883	- .780005	- .411955	.70
.80	-2.088004	-1.664529	-1.236393	- .828287	- .456215	- .129776	.80
.90	-1.809510	-1.327212	- .885638	- .497116	- .166914	.104257	.90
1.00	-1.451824	- .956507	- .537636	- .194147	.078882	.288242	1.00
1.10	-1.045802	- .579507	- .212395	.068637	.276614	.423568	1.10
1.20	- .623011	- .220265	.074750	.283975	.425507	.514034	1.20
1.30	- .212999	.101683	.313476	.448891	.527781	.565021	1.30
1.40	.158975	.372194	.498348	.564042	.587834	.582781	1.40
1.50	.473549	.582793	.628327	.632916	.611467	.573849	1.50
1.60	.718128	.730424	.706022	.661016	.605202	.544590	1.60
1.70	.887018	.816786	.736813	.655060	.575716	.500885	1.70
1.80	.980903	.847379	.727921	.622290	.529406	.447918	1.80
1.90	1.005825	.830419	.687550	.569897	.472079	.390073	1.90
2.00	.971838	.775717	.624121	.504581	.408762	.330901	2.00
2.10	.891503	.693655	.539812	.432255	.343601	.273145	2.10
2.20	.778443	.594315	.459374	.357869	.279854	.218807	2.20
2.30	.646042	.486812	.371323	.285342	.219924	.169238	2.30
2.40	.506424	.378852	.286324	.217579	.165452	.125239	2.40
2.50	.369738	.276486	.207918	.156549	.117416	.087165	2.50
2.60	.243764	.184060	.138447	.103403	.076253	.055028	2.60
2.70	.133814	.104303	.079202	.058616	.041973	.028582	2.70
2.80	.042873	.038519	.030597	.022127	.014271	.007408	2.80
2.90	-.028105	-.013156	-.007636	-.006517	-.007374	-.009025	2.90
3.00	-.079773	-.051472	-.036256	-.028035	-.023621	-.021303	3.00
3.10	-.113952	-.077795	-.056342	-.043304	-.035192	-.030026	3.10
3.20	-.133185	-.093851	-.069144	-.053270	-.042824	-.035779	3.20
3.30	-.140362	-.101518	-.075965	-.058881	-.047229	-.039110	3.30
3.40	-.138396	-.102660	-.078070	-.061034	-.049067	-.040516	3.40
3.50	-.130002	-.099010	-.076622	-.060545	-.048929	-.040438	3.50
3.60	-.117547	-.092091	-.072649	-.058132	-.047331	-.039253	3.60
3.70	-.102971	-.083181	-.067016	-.054399	-.044709	-.037282	3.70
3.80	-.087768	-.073300	-.060432	-.049848	-.041420	-.034789	3.80
3.90	-.073016	-.063224	-.053446	-.044873	-.037754	-.031983	3.90
4.00	-.059421	-.053504	-.046473	-.039778	-.033935	-.029033	4.00
4.10	-.047385	-.044506	-.039802	-.034788	-.030132	-.026066	4.10
4.20	-.037077	-.036442	-.033627	-.030057	-.026465	-.023173	4.20
4.30	-.028497	-.029405	-.028056	-.025686	-.023019	-.020423	4.30
4.40	-.021533	-.023405	-.023139	-.021731	-.019844	-.017856	4.40
4.50	-.016008	-.018390	-.018880	-.018216	-.016969	-.015501	4.50
4.60	-.011715	-.014274	-.015250	-.015138	-.014402	-.013367	4.60
4.70	-.008445	-.010951	-.012201	-.012480	-.012138	-.011456	4.70
4.80	-.005998	-.008308	-.009674	-.010210	-.010164	-.009763	4.80
4.90	-.004199	-.006235	-.007605	-.008294	-.008459	-.008277	4.90

t	SKEWNESS .6	.7	.8	.9	1.0	1.1	t
.00	−2.626655	−2.478942	−2.115071	−1.542070	− .781467	.129293	.00
.10	−2.283191	−1.984713	−1.526917	− .933664	− .238010	.518231	.10
.20	−1.886598	−1.496838	−1.005251	− .441900	.159245	.761246	.20
.30	−1.471780	−1.041901	− .561462	− .059315	.435740	.895647	.30
.40	−1.066503	− .637251	− .198726	.225751	.615190	.951124	.40
.50	− .691263	− .292484	.085577	.426932	.718714	.950936	.50
.60	− .359748	− .011045	.297857	.558300	.764493	.913026	.60
.70	− .225920	.208228	.446717	.633354	.767747	.851009	.70
.80	.146078	.369701	.541745	.664415	.740914	.775038	.80
.90	.318539	.479757	.592641	.652309	.693932	.692521	.90
1.00	.441406	.545840	.608622	.636249	.634561	.608726	1.00
1.10	.520183	.575713	.598052	.593860	.568717	.577633	1.10
1.20	.561361	.576925	.568240	.541282	.500789	.450516	1.20
1.30	.571777	.556451	.525374	.483328	.433925	.379892	1.30
1.40	.558132	.520476	.474525	.423655	.370284	.316137	1.40
1.50	.526650	.474288	.419728	.364948	.311256	.259494	1.50
1.60	.482864	.422255	.364084	.309088	.257638	.209867	1.60
1.70	.431504	.367861	.309884	.257311	.209788	.166927	1.70
1.80	.376465	.313781	.258737	.210344	.167746	.130204	1.80
1.90	.320823	.261975	.211694	.168529	.131325	.099148	1.90
2.00	.266905	.213795	.169358	.131919	.100192	.073173	2.00
2.10	.216372	.170089	.131987	.100359	.073921	.051692	2.10
2.20	.170315	.131304	.099584	.073557	.052040	.034138	2.20
2.30	.129359	.097575	.071962	.051132	.034060	.019978	2.30
2.40	.093758	.068803	.048813	.032656	.019500	.008722	2.40
2.50	.063480	.044730	.029747	.017680	.007898	−.000673	2.50
2.60	.038290	.024986	.014337	.005763	−.001173	−.006804	2.60
2.70	.017813	.009140	.002140	−.003520	−.008104	−.011818	2.70
2.80	.001587	−.003268	−.007278	−.010566	−.013247	−.015421	2.80
2.90	−.010892	−.012701	−.014329	−.015735	−.016912	−.017873	2.90
3.00	−.020139	−.019603	−.019397	−.019351	−.019371	−.019401	3.00
3.10	−.026654	−.024392	−.022826	−.021702	−.020858	−.020195	3.10
3.20	−.030906	−.027448	−.024896	−.023035	−.021575	−.020415	3.20
3.30	−.033325	−.029108	−.025964	−.023565	−.021691	−.020194	3.30
3.40	−.034291	−.029671	−.026174	−.023474	−.021349	.019643	3.40
3.50	−.034138	−.029390	−.025751	−.022914	−.020666	−.018853	3.50
3.60	−.033150	−.028481	−.024861	−.022012	−.019739	−.017898	3.60
3.70	−.031565	−.027125	−.023639	−.020872	−.018646	−.016835	3.70
3.80	−.029578	−.025466	−.022198	−.019576	−.017452	−.015712	3.80
3.90	−.027346	−.023624	−.020624	−.018192	−.016204	−.014566	3.90
4.00	−.024995	−.021691	−.018989	−.016772	−.014943	−.013424	4.00
4.10	−.022619	−.019739	−.017345	−.015355	−.013696	−.012307	4.10
4.20	−.020290	−.017822	−.015732	−.013971	−.012486	−.011232	4.20
4.30	−.018059	−.015979	−.014182	−.012643	−.011329	−.010208	4.30
4.40	−.015959	−.014236	−.012712	−.011384	−.010235	−.009243	4.40
4.50	−.014014	−.012611	−.011337	−.010205	−.009210	−.008341	4.50
4.60	−.012233	−.011113	−.010064	−.009111	−.008258	−.007504	4.60
4.70	−.010621	−.009747	−.008897	−.008104	−.007381	−.006732	4.70
4.80	−.009176	−.008511	−.007834	−.007183	−.006577	−.006023	4.80
4.90	−.007890	−.007401	−.006873	−.006347	−.005844	−.005377	4.90

t	SKEWNESS						t
	.0	.1	.2	.3	.4	.5	
5.00	-.002899	-.004632	-.005930	-.006692	-.007000	-.006981	5.00
5.10	-.001974	-.003406	-.004587	-.005365	-.005761	-.005861	5.10
5.20	-.001326	-.002481	-.003522	-.004275	-.004717	-.004899	5.20
5.30	-.000879	-.001790	-.002685	-.003386	-.003843	-.004078	5.30
5.40	-.000575	-.001279	-.002032	-.002667	-.003117	-.003381	5.40
5.50	-.000372	-.000906	-.001528	-.002090	-.002516	-.002793	5.50
5.60	-.000237	-.000637	-.001141	-.001629	-.002023	-.002298	5.60
5.70	-.000149	-.000443	-.000847	-.001263	-.001620	-.001885	5.70
5.80	-.000093	-.000306	-.000625	-.000972	-.001292	-.001541	5.80
5.90	-.000057	-.000210	-.000458	-.000749	-.001026	-.001256	5.90
6.00	-.000035	-.000142	-.000334	-.000572	-.000812	-.001020	6.00
6.10		-.000096	-.000242	-.000436	-.000641	-.000826	6.10
6.20		-.000064	-.000174	-.000330	-.000504	-.000668	6.20
6.30		-.000043	-.000125	-.000249	-.000395	-.000538	6.30
6.40		-.000028	-.000089	-.000188	-.000308	-.000432	6.40
6.50		-.000018	-.000063	-.000140	-.000240	-.000346	6.50
6.60		-.000012	-.000045	-.000105	-.000186	-.000277	6.60
6.70			-.000031	-.000078	-.000144	-.000221	6.70
6.80			-.000022	-.000058	-.000111	-.000176	6.80
6.90			-.000015	-.000043	-.000086	-.000140	6.90
7.00			-.000011	-.000031	-.000066	-.000111	7.00
7.10			-.000007	-.000023	-.000050	-.000087	7.10
7.20			-.000005	-.000017	-.000038	-.000069	7.20
7.30			-.000003	-.000012	-.000029	-.000054	7.30
7.40			-.000002	-.000009	-.000022	-.000043	7.40
7.50				-.000006	-.000017	-.000034	7.50
7.60				-.000005	-.000013	-.000026	7.60
7.70				-.000003	-.000010	-.000021	7.70
7.80				-.000002	-.000007	-.000016	7.80
7.90				-.000002	-.000005	-.000012	7.90
8.00				-.000001	-.000004	-.000010	8.00
8.10					-.000003	-.000007	8.10
8.20					-.000002	-.000006	8.20
8.30					-.000002	-.000005	8.30
8.40					-.000001	-.000004	8.40
8.50						-.000003	8.50
8.60						-.000002	8.60
8.70						-.000002	8.70
8.80						-.000001	8.80
8.90						-.000001	8.90
9.00						-.000001	9.00
9.10							9.10
9.20							9.20
9.30							9.30
9.40							9.40
9.50							9.50
9.60							9.60
9.70							9.70
9.80							9.80
9.90							9.90

t	SKEWNESS .6	.7	.8	.9	1.0	1.1	t
5.00	-.006755	-.006411	-.006010	-.005592	-.005180	-.004788	5.00
5.10	-.005760	-.005534	-.005239	-.004913	-.004580	-.004256	5.10
5.20	-.004893	-.004761	-.004553	-.004305	-.004040	-.003775	5.20
5.30	-.004141	-.004083	-.003946	-.003763	-.003557	-.003343	5.30
5.40	-.003493	-.003491	-.003411	-.003282	-.003125	-.002955	5.40
5.50	-.002937	-.002977	-.002942	-.002857	-.002741	-.002608	5.50
5.60	-.002461	-.002531	-.002531	-.002482	-.002400	-.002298	5.60
5.70	-.002057	-.002147	-.002173	-.002151	-.002097	-.002022	5.70
5.80	-.001714	-.001817	-.001862	-.001862	-.001830	-.001777	5.80
5.90	-.001425	-.001534	-.001592	-.001609	-.001595	-.001559	5.90
6.00	-.001181	-.001292	-.001358	-.001387	-.001388	-.001367	6.00
6.10	-.000977	-.001086	-.001157	-.001195	-.001206	-.001197	6.10
6.20	-.000806	-.000911	-.000984	-.001027	-.001046	-.001047	6.20
6.30	-.000663	-.000763	-.000835	-.000882	-.000907	-.000914	6.30
6.40	-.000545	-.000638	-.000708	-.000756	-.000785	-.000798	6.40
6.50	-.000447	-.000532	-.000599	-.000648	-.000679	-.000696	6.50
6.60	-.000365	-.000443	-.000506	-.000554	-.000587	-.000606	6.60
6.70	-.000298	-.000368	-.000427	-.000473	-.000506	-.000528	6.70
6.80	-.000243	-.000306	-.000360	-.000404	-.000437	-.000459	6.80
6.90	-.000198	-.000254	-.000303	-.000344	-.000376	-.000399	6.90
7.00	-.000160	-.000210	-.000255	-.000293	-.000324	-.000346	7.00
7.10	-.000130	-.000173	-.000214	-.000249	-.000278	-.000300	7.10
7.20	-.000105	-.000143	-.000180	-.000212	-.000239	-.000261	7.20
7.30	-.000085	-.000118	-.000150	-.000180	-.000205	-.000226	7.30
7.40	-.000069	-.000097	-.000126	-.000153	-.000176	-.000195	7.40
7.50	-.000055	-.000080	-.000105	-.000129	-.000151	-.000169	7.50
7.60	-.000044	-.000066	-.000088	-.000109	-.000129	-.000146	7.60
7.70	-.000036	-.000054	-.000073	-.000093	-.000111	-.000126	7.70
7.80	-.000029	-.000044	-.000061	-.000078	-.000095	-.000109	7.80
7.90	-.000023	-.000036	-.000051	-.000066	-.000081	-.000094	7.90
8.00	-.000018	-.000029	-.000042	-.000056	-.000069	-.000081	8.00
8.10	-.000015	-.000024	-.000035	-.000047	-.000059	-.000070	8.10
8.20	-.000012	-.000020	-.000029	-.000040	-.000050	-.000060	8.20
8.30	-.000009	-.000016	-.000024	-.000033	-.000043	-.000052	8.30
8.40	-.000007	-.000013	-.000020	-.000028	-.000037	-.000045	8.40
8.50	-.000006	-.000011	-.000017	-.000024	-.000031	-.000039	8.50
8.60	-.000005	-.000009	-.000014	-.000020	-.000026	-.000033	8.60
8.70	-.000004	-.000007	-.000011	-.000017	-.000023	-.000029	8.70
8.80	-.000003	-.000006	-.000009	-.000014	-.000019	-.000025	8.80
8.90	-.000002	-.000005	-.000008	-.000012	-.000016	-.000021	8.90
9.00	-.000002	-.000004	-.000006	-.000010	-.000014	-.000018	9.00
9.10	-.000002	-.000003	-.000005	-.000008	-.000012	-.000016	9.10
9.20	-.000001	-.000002	-.000004	-.000007	-.000010	-.000013	9.20
9.30	-.000001	-.000002	-.000004	-.000006	-.000008	-.000011	9.30
9.40	-.000001	-.000002	-.000003	-.000005	-.000007	-.000010	9.40
9.50	-.000001	-.000001	-.000002	-.000004	-.000006	-.000008	9.50
9.60		-.000001	-.000002	-.000003	-.000005	-.000007	9.60
9.70		-.000001	-.000002	-.000003	-.000004	-.000006	9.70
9.80		-.000001	-.000001	-.000002	-.000004	-.000005	9.80
9.90		-.000001	-.000001	-.000002	-.000003	-.000005	9.90

t	SKEWNESS						t
	.0	.1	.2	.3	.4	.5	
10.00							10.00
10.10							10.10
10.20							10.20
10.30							10.30
10.40							10.40
10.50							10.50
10.60							10.60
10.70							10.70
10.80							10.80
10.90							10.90
11.00							11.00
11.10							11.10
11.20							11.20
11.30							11.30
11.40							11.40
11.50							11.50
11.60							11.60
11.70							11.70
11.80							11.80
11.90							11.90
12.00							12.00
12.10							12.10
12.20							12.20
12.30							12.30
12.40							12.40
12.50							12.50
12.60							12.60
12.70							12.70
12.80							12.80
12.90							12.90
13.00							13.00
13.10							13.10
13.20							13.20
13.30							13.30
13.40							13.40
13.50							13.50
13.60							13.60
13.70							13.70
13.80							13.80
13.90							13.90
14.00							14.00
14.10							14.10
14.20							14.20
14.30							14.30
14.40							14.40
14.50							14.50
14.60							14.60
14.70							14.70
14.80							14.80
14.90							14.90

t	SKEWNESS .6	.7	.8	.9	1.0	1.1	t
10.00			-.000001	-.000002	-.000003	-.000004	10.00
10.10			-.000001	-.000001	-.000002	-.000003	10.10
10.20			-.000001	-.000001	-.000002	-.000003	10.20
10.30			-.000001	-.000001	-.000002	-.000002	10.30
10.40				-.000001	-.000001	-.000002	10.40
10.50				-.000001	-.000001	-.000002	10.50
10.60				-.000001	-.000001	-.000002	10.60
10.70					-.000001	-.000001	10.70
10.80					-.000001	-.000001	10.80
10.90					-.000001	-.000001	10.90
11.00						-.000001	11.00
11.10						-.000001	11.10
11.20						-.000001	11.20
11.30						-.000001	11.30
11.40							11.40
11.50							11.50
11.60							11.60
11.70							11.70
11.80							11.80
11.90							11.90
12.00							12.00
12.10							12.10
12.20							12.20
12.30							12.30
12.40							12.40
12.50							12.50
12.60							12.60
12.70							12.70
12.80							12.80
12.90							12.90
13.00							13.00
13.10							13.10
13.20							13.20
13.30							13.30
13.40							13.40
13.50							13.50
13.60							13.60
13.70							13.70
13.80							13.80
13.90							13.90
14.00							14.00
14.10							14.10
14.20							14.20
14.30							14.30
14.40							14.40
14.50							14.50
14.60							14.60
14.70							14.70
14.80							14.80
14.90							14.90

	SKEWNESS						
t	.0	.1	.2	.3	.4	.5	t
-9.90							-9.90
-9.80							-9.80
-9.70							-9.70
-9.60							-9.60
-9.50							-9.50
-9.40							-9.40
-9.30							-9.30
-9.20							-9.20
-9.10							-9.10
-9.00							-9.00
-8.90							-8.90
-8.80							-8.80
-8.70							-8.70
-8.60							-8.60
-8.50							-8.50
-8.40							-8.40
-8.30							-8.30
-8.20							-8.20
-8.10							-8.10
-8.00							-8.00
-7.90							-7.90
-7.80							-7.80
-7.70							-7.70
-7.60							-7.60
-7.50							-7.50
-7.40							-7.40
-7.30							-7.30
-7.20							-7.20
-7.10							-7.10
-7.00							-7.00
-6.90							-6.90
-6.80							-6.80
-6.70							-6.70
-6.60							-6.60
-6.50							-6.50
-6.40							-6.40
-6.30							-6.30
-6.20							-6.20
-6.10							-6.10
-6.00	.000175						-6.00
-5.90	.000282						-5.90
-5.80	.000447						-5.80
-5.70	.000700						-5.70
-5.60	.001081						-5.60
-5.50	.001648	.000447					-5.50
-5.40	.002477	.000786					-5.40
-5.30	.003672	.001349					-5.30
-5.20	.005364	.002265					-5.20
-5.10	.007720	.003718					-5.10
-5.00	.010942	.005964	.001401				-5.00

	SKEWNESS						
t	.6	.7	.8	.9	1.0	1.1	t
-9.90							-9.90
-9.80							-9.80
-9.70							-9.70
-9.60							-9.60
-9.50							-9.50
-9.40							-9.40
-9.30							-9.30
-9.20							-9.20
-9.10							-9.10
-9.00							-9.00
-8.90							-8.90
-8.80							-8.80
-8.70							-8.70
-8.60							-8.60
-8.50							-8.50
-8.40							-8.40
-8.30							-8.30
-8.20							-8.20
-8.10							-8.10
-8.00							-8.00
-7.90							-7.90
-7.80							-7.80
-7.70							-7.70
-7.60							-7.60
-7.50							-7.50
-7.40							-7.40
-7.30							-7.30
-7.20							-7.20
-7.10							-7.10
-7.00							-7.00
-6.90							-6.90
-6.80							-6.80
-6.70							-6.70
-6.60							-6.60
-6.50							-6.50
-6.40							-6.40
-6.30							-6.30
-6.20							-6.20
-6.10							-6.10
-6.00							-6.00
-5.90							-5.90
-5.80							-5.80
-5.70							-5.70
-5.60							-5.60
-5.50							-5.50
-5.40							-5.40
-5.30							-5.30
-5.20							-5.20
-5.10							-5.10
-5.00							-5.00

t	SKEWNESS						t
	.0	.1	.2	.3	.4	.5	
–4.90	.015267	.009347	.002766				–4.90
–4.80	.020954	.014312	.005236				–4.80
–4.70	.028274	.021396	.009515				–4.70
–4.60	.037474	.031216	.016605				–4.60
–4.50	.048736	.044414	.027840				–4.50
–4.40	.062116	.061567	.044852	.013863			–4.40
–4.30	.077464	.083049	.069421	.029156			–4.30
–4.20	.094326	.108841	.103173	.056718			–4.20
–4.10	.111837	.138280	.147080	.102430			–4.10
–4.00	.128611	.169792	.200769	.172160	.045017		–4.00
–3.90	.142641	.200610	.261691	.269650	.117666		–3.90
–3.80	.151238	.226551	.324272	.393475	.259945		–3.80
–3.70	.151024	.241913	.379259	.533619	.495046		–3.70
–3.60	.138020	.239563	.413503	.668647	.822975		–3.60
–3.50	.107844	.211314	.410484	.764762	1.200746	.742856	–3.50
–3.40	.056066	.148638	.351775	.778218	1.531759	1.764469	–3.40
–3.30	–.021297	.043742	.219572	.661208	1.674232	3.080073	–3.30
–3.20	–.127125	–.109010	.000159	.372046	1.472443	4.108961	–3.20
–3.10	–.262416	–.311448	–.312050	–.112822	.805005	4.095235	–3.10
–3.00	–.425457	–.559902	–.710518	–.786892	–.364035	2.484221	–3.00
–2.90	–.611017	–.843721	–1.173459	–1.605106	–1.949117	–.757276	–2.90
–2.80	–.809701	–1.144331	–1.662462	–2.481732	–3.736776	–5.045593	–2.80
–2.70	–1.007607	–1.435140	–2.123635	–3.296576	–5.414832	–9.343178	–2.70
–2.60	–1.186451	–1.682587	–2.491694	–3.909747	–6.720425	–12.49085	–2.60
–2.50	–1.324208	–1.848487	–2.696991	–4.183196	–7.073686	–13.56574	–2.50
–2.40	–1.396544	–1.893676	–2.674851	–4.005451	–6.533877	–12.13850	–2.40
–2.30	–1.378834	–1.782727	–2.376055	–3.314841	–4.966528	–8.363173	–2.30
–2.20	–1.248851	–1.489294	–1.776879	–2.116465	–2.502962	–2.900863	–2.20
–2.10	–.989868	–1.001128	–.886935	–.489062	.567018	3.273070	–2.10
–2.00	–.593901	–.326029	.246817	1.420125	3.844994	9.107213	–2.00
–1.90	–.064672	.508204	1.541012	3.412172	6.893098	13.691811	–1.90
–1.80	.580144	1.450348	2.882212	5.262389	9.306193	16.410943	–1.80
–1.70	1.307853	2.429501	4.138184	6.751069	10.775134	17.010500	–1.70
–1.60	2.071248	3.360003	5.172815	7.693951	11.128417	15.586539	–1.60
–1.50	2.810937	4.149259	5.862695	7.957495	10.348409	12.514059	–1.50
–1.40	3.459534	4.707433	6.113044	7.525126	8.562045	8.342962	–1.40
–1.30	3.947529	4.957908	5.870642	6.402253	6.010840	3.687149	–1.30
–1.20	4.210340	4.847185	5.131842	4.709910	3.007841	–.872959	–1.20
–1.10	4.195847	4.352868	3.786317	2.618573	–.109997	–4.861982	–1.10
–1.00	3.871531	3.488518	2.403342	.335208	–3.026213	–7.950511	–1.00
– .90	3.230259	2.304551	.639885	–1.922775	–5.480152	–9.966922	– .90
– .80	2.293820	.884850	–1.192420	–3.952789	–7.289797	–10.88572	– .80
– .70	1.113544	–.660680	–2.935015	–5.588315	–8.361193	–10.80001	– .70
– .60	–.232372	–2.206781	–4.441265	–6.713777	–8.685678	–9.885928	– .60
– .50	–1.644805	–3.625505	–5.591378	–7.271912	–8.327546	–8.365606	– .50
– .40	–3.012214	–4.799309	–6.303511	–7.263647	–7.405560	–6.473884	– .40
– .30	–4.222257	–5.632987	–6.364810	–6.741491	–6.071759	–4.431674	– .30
– .20	–5.171124	–6.063022	–6.308038	–5.798132	–4.490653	–2.427427	– .20
– .10	–5.776254	–6.063275	–5.656056	–4.552304	–2.821126	–.606621	– .10
.00	–5.984135	–5.646455	–4.665204	–3.133923	–1.202514	.931715	.00

t	SKEWNESS .6	.7	.8	.9	1.0	1.1	t
-4.90							-4.90
-4.80							-4.80
-4.70							-4.70
-4.60							-4.60
-4.50							-4.50
-4.40							-4.40
-4.30							-4.30
-4.20							-4.20
-4.10							-4.10
-4.00							-4.00
-3.90							-3.90
-3.80							-3.80
-3.70							-3.70
-3.60							-3.60
-3.50							-3.50
-3.40							-3.40
-3.30							-3.30
-3.20	26.875960						-3.20
-3.10	8.895339						-3.10
-3.00	14.844781						-3.00
-2.90	11.870834						-2.90
-2.80	.511734						-2.80
-2.70	-14.54133	49.586240					-2.70
-2.60	-26.74182	-50.43212					-2.60
-2.50	-31.55201	-100.9999					-2.50
-2.40	-27.79824	-93.03413	-897.1775				-2.40
-2.30	-17.22889	-51.64289	-371.6563				-2.30
-2.20	-3.200061	-3.279440	-6.531180	14261.298			-2.20
-2.10	10.702406	35.142528	156.15605	2561.9104			-2.10
-2.00	21.648976	56.825503	190.10374	1209.5822			-2.00
-1.90	28.031135	62.079701	138.10092	540.76774	-1793.994		-1.90
-1.80	29.481130	54.921276	106.57407	232.57201	-1241.483	-21843131	-1.80
-1.70	26.608535	40.473714	53.588006	5.612894	-846.0655	-27894.86	-1.70
-1.60	20.618004	23.393943	11.107951	-75.76662	-565.6321	-3569.339	-1.60
-1.50	12.932907	7.147980	-17.91647	-102.2365	-368.8605	-953.1063	-1.50
-1.40	4.898190	-6.141522	-34.40810	-100.0908	-232.5508	-304.3152	-1.40
-1.30	-2.409460	-15.52843	-40.94149	-84.79357	-139.5989	-81.95704	-1.30
-1.20	-8.267851	-20.94228	-40.46917	-65.13526	-77.45935	4.106084	-1.20
-1.10	-12.31332	-22.86993	-35.67140	-45.81429	-36.98463	36.888914	-1.10
-1.00	-14.49033	-22.08158	-28.68817	-29.06763	-11.54861	46.646847	-1.00
- .90	-14.97304	-19.42548	-21.06770	-15.70089	3.611887	46.164258	- .90
- .80	-14.08005	-15.69277	-13.82424	-5.732500	11.890584	41.340786	- .80
- .70	-12.19660	-11.54227	-7.541844	1.210768	15.680242	35.022093	- .70
- .60	-9.712425	-7.470641	-2.484734	5.659817	16.646714	28.587090	- .60
- .50	-6.978085	-3.813356	1.301813	8.169192	15.931861	22.674434	- .50
- .40	-4.279153	-.763934	3.909282	9.248079	14.303076	17.539966	- .40
- .30	-1.825508	1.598081	5.503477	9.329346	12.262853	13.241811	- .30
- .20	.247988	3.277084	6.280186	8.761196	10.128576	9.739410	- .20
- .10	1.873235	4.333462	6.435881	7.811120	8.090226	6.947384	.10
.00	3.036428	4.859952	6.150084	6.675819	6.251738	4.764787	.00

	SKEWNESS						
t	.0	.1	.2	.3	.4	.5	t
.00	-5.984135	-5.646455	-4.665204	-3.133923	-1.202514	.931715	.00
.10	-5.776254	-4.861356	-3.438799	-1.670337	.254500	2.132985	.10
.20	-5.171124	-3.786395	-2.090353	– .274980	1.472275	2.981242	.20
.30	-4.222257	-2.520401	– .731916	.960727	2.406462	3.491248	.30
.40	-3.012214	-1.171892	.536015	1.971948	3.043064	3.699785	.40
.50	-1.644805	.151841	1.632165	2.721665	3.393319	3.657272	.50
.60	– .232372	1.354605	2.499227	3.199050	3.487452	3.420390	.60
.70	1.113544	2.359129	3.105617	3.415727	3.368150	3.046137	.70
.80	2.293820	3.112206	3.444507	3.400697	3.084467	2.587429	.80
.90	3.230259	3.586870	3.530678	3.194705	2.686604	2.090218	.90
1.00	3.871531	3.781737	3.395867	2.844710	2.221834	1.591942	1.00
1.10	4.195847	3.717924	3.083342	2.398951	1.731648	1.121050	1.10
1.20	4.210340	3.434203	2.642373	1.902952	1.250054	.697352	1.20
1.30	3.947529	2.981144	2.123169	1.396604	.802892	.332911	1.30
1.40	3.459534	2.414999	1.572664	.912359	.407952	.033260	1.40
1.50	2.810937	1.792005	1.031410	.474437	.075673	–.201246	1.50
1.60	2.071248	1.163611	.531626	.098868	–.189775	–.374091	1.60
1.70	1.307853	.572967	.096362	–.205823	–.389234	–.491358	1.70
1.80	.580144	.052784	–.260397	–.437511	–.527257	–.560670	1.80
1.90	–.064672	–.375458	–.532887	–.599134	–.610882	–.590313	1.90
2.00	–.593901	–.701132	–.722234	–.697299	–.648554	–.588568	2.00
2.10	–.989868	–.913199	–.809084	–.740958	–.649236	–.563231	2.10
2.20	–1.248851	–1.048345	–.880804	–.740237	–.621724	–.521293	2.20
2.30	–1.378834	–1.088823	–.872200	–.705481	–.574151	–.468771	2.30
2.40	–1.396544	–1.060287	–.821890	–.646523	–.513679	–.410636	2.40
2.50	–1.324208	–.979805	–.742355	–.572181	–.446329	–.350831	2.50
2.60	–1.186451	–.864201	–.644933	–.489957	–.376936	–.292338	2.60
2.70	–1.007607	–.728816	–.539310	–.405909	–.309189	–.237290	2.70
2.80	–.809701	–.586683	–.433276	–.324649	–.245729	–.187092	2.80
2.90	–.611017	–.448101	–.332680	–.249447	–.188292	–.142554	2.90
3.00	–.425457	–.320545	–.241545	–.182391	–.137856	–.104005	3.00
3.10	–.262416	–.208830	–.162278	–.124581	–.094796	–.071429	3.10
3.20	–.127125	–.115450	–.095950	–.076334	–.059035	–.044545	3.20
3.30	–.021297	–.041016	–.042586	–.037380	–.030168	–.022905	3.30
3.40	.056066	.015276	–.001451	–.007041	–.007579	–.005958	3.40
3.50	.107844	.055191	.028687	.015614	.009469	.006895	3.50
3.60	.138020	.081059	.049341	.031649	.021765	.016260	3.60
3.70	.151024	.095440	.062130	.042162	.030093	.022718	3.70
3.80	.151238	.100881	.068649	.048214	.035196	.026809	3.80
3.90	.142641	.099734	.070375	.050783	.037750	.029018	3.90
4.00	.128611	.094057	.068614	.050736	.038353	.029771	4.00
4.10	.111837	.085556	.064474	.048815	.037518	.029431	4.10
4.20	.094326	.075579	.058859	.045636	.035673	.028304	4.20
4.30	.077464	.065141	.052475	.041691	.033167	.026640	4.30
4.40	.062116	.054955	.045856	.037366	.030277	.024639	4.40
4.50	.048736	.045489	.039380	.032946	.027214	.022458	4.50
4.60	.037474	.037012	.033302	.028639	.024137	.020216	4.60
4.70	.028274	.029644	.027773	.024583	.021160	.018002	4.70
4.80	.020954	.023399	.022872	.020863	.018359	.015877	4.80
4.90	.015267	.018220	.018617	.017524	.015780	.013884	4.90

t	SKEWNESS .6	.7	.8	.9	1.0	1.1	t
.00	3.036428	4.859952	6.150084	6.675819	6.251738	4.764787	.00
.10	3.764079	4.963167	5.576252	5.493291	4.660385	3.090633	.10
.20	4.109449	4.750215	4.838542	4.354939	3.327388	1.831652	.20
.30	4.140621	4.319865	4.032352	3.316558	2.242176	.905655	.30
.40	3.930837	3.757417	3.227075	2.407733	1.382071	.203484	.40
.50	3.551296	3.132436	2.469942	1.639509	.718714	-.216483	.50
.60	3.066280	2.498523	1.790194	1.010443	.222193	-.519178	.60
.70	2.500095	1.894408	1.203125	.511222	-.136437	-.704349	.70
.80	1.986914	1.345841	.713700	.128065	-.383710	-.802822	.80
.90	1.468665	.867818	.319643	-.154839	-.543055	-.838819	.90
1.00	.997981	.466872	.013947	-.353672	-.634561	-.831126	1.00
1.10	.588480	.143223	-.213162	-.483906	-.675048	-.799963	1.10
1.20	.246555	-.107330	-.372754	-.559657	-.678305	-.738617	1.20
1.30	-.027004	-.291823	-.476136	-.593360	-.655415	-.672661	1.30
1.40	-.235490	-.418856	-.534137	-.595648	-.615113	-.602070	1.40
1.50	-.384920	-.497630	-.556659	-.575384	-.564151	-.530962	1.50
1.60	-.482913	-.537238	-.552429	-.539782	-.507631	-.462145	1.60
1.70	-.537788	-.546198	-.528908	-.494574	-.449308	-.397442	1.70
1.80	-.557885	-.532154	-.492291	-.444204	-.391863	-.337936	1.80
1.90	-.551088	-.501728	-.447585	-.392019	-.322869	-.284173	1.90
2.00	-.524515	-.460475	-.398722	-.340461	-.286261	-.236315	2.00
2.10	-.484347	-.412908	-.348694	-.291231	-.239949	-.194255	2.10
2.20	-.435762	-.362581	-.299695	-.245447	-.198488	-.157714	2.20
2.30	-.382945	-.312187	-.253259	-.203769	-.161911	-.126298	2.30
2.40	-.329152	-.263683	-.210383	-.166512	-.130066	-.099556	2.40
2.50	-.276808	-.218407	-.171645	-.133735	-.102677	-.077009	2.50
2.60	-.227620	-.177197	-.137300	-.105316	-.079392	-.058182	2.60
2.70	-.182699	-.140495	-.107364	-.081012	-.059819	-.042614	2.70
2.80	-.142668	-.108440	-.081682	-.060501	-.043554	-.029872	2.80
2.90	-.107778	-.080951	-.059987	-.043420	-.030199	-.019560	2.90
3.00	-.077994	-.057791	-.041940	-.029389	-.019371	-.011318	3.00
3.10	-.053083	-.038624	-.027165	-.018032	-.010715	-.004823	3.10
3.20	-.032678	-.023053	-.015273	-.008988	-.003906	.000208	3.20
3.30	-.016331	-.010658	-.005883	-.001918	.001348	.004024	3.30
3.40	-.003556	.001017	.001367	.003485	.005306	.006839	3.40
3.50	.006139	.006275	.006814	.007498	.008195	.008839	3.50
3.60	.013228	.011598	.010761	.010367	.010212	.010180	3.60
3.70	.018156	.015297	.013480	.012305	.011527	.010996	3.70
3.80	.021325	.017679	.015207	.013496	.012283	.011399	3.80
3.90	.023094	.019012	.016148	.014099	.012602	.011482	3.90
4.00	.023772	.019527	.016476	.014245	.012583	.011321	4.00
4.10	.023625	.019421	.016338	.014045	.012312	.010980	4.10
4.20	022874	.018856	.015856	.013591	.011857	.010510	4.20
4.30	.021701	.017966	.015129	.012956	.011272	.009952	4.30
4.40	.020251	.016860	.014238	.012200	.010603	.009339	4.40
4.50	.018642	.015623	.013246	.011372	.009885	.008697	4.50
4.60	.016962	.014323	.012204	.010508	.009145	.008046	4.60
4.70	.015279	.013010	.011150	.009636	.008405	.007401	4.70
4.80	.013642	.011722	.010112	.008779	.007679	.006773	4.80
4.90	.012085	.010486	.009111	.007951	.006980	.006170	4.90

t	SKEWNESS .0	.1	.2	.3	.4	.5	t
5.00	.010942	.014005	.014991	.014581	.013449	.012048	5.00
5.10	.007720	.010635	.011950	.012026	.011373	.010381	5.10
5.20	.005364	.007983	.009436	.009839	.009549	.008888	5.20
5.30	.003672	.005926	.007384	.007988	.007964	.007564	5.30
5.40	.002477	.004352	.005729	.006439	.006601	.006402	5.40
5.50	.001648	.003164	.004409	.005155	.005439	.005391	5.50
5.60	.001081	.002277	.003367	.004101	.004457	.004518	5.60
5.70	.000700	.001623	.002552	.003242	.003634	.003769	5.70
5.80	.000447	.001146	.001920	.002543	.002948	.003130	5.80
5.90	.000282	.000802	.001435	.001992	.002380	.002590	5.90
6.00	.000175	.000556	.001065	.001549	.001913	.002134	6.00
6.10		.000382	.000786	.001198	.001531	.001753	6.10
6.20		.000261	.000576	.000922	.001220	.001434	6.20
6.30		.000176	.000420	.000706	.000969	.001170	6.30
6.40		.000118	.000304	.000538	.000766	.000951	6.40
6.50		.000079	.000219	.000409	.000604	.000771	6.50
6.60		.000052	.000157	.000309	.000474	.000623	6.60
6.70			.000112	.000232	.000371	.000502	6.70
6.80			.000079	.000174	.000290	.000403	6.80
6.90			.000056	.000130	.000225	.000323	6.90
7.00			.000039	.000097	.000175	.000259	7.00
7.10			.000027	.000072	.000135	.000206	7.10
7.20			.000019	.000053	.000104	.000164	7.20
7.30			.000013	.000039	.000080	.000130	7.30
7.40			.000009	.000029	.000061	.000103	7.40
7.50				.000021	.000047	.000082	7.50
7.60				.000015	.000036	.000064	7.60
7.70				.000011	.000027	.000051	7.70
7.80				.000008	.000021	.000040	7.80
7.90				.000006	.000016	.000031	7.90
8.00				.000004	.000012	.000024	8.00
8.10					.000009	.000019	8.10
8.20					.000007	.000015	8.20
8.30					.000005	.000012	8.30
8.40					.000004	.000009	8.40
8.50						.000007	8.50
8.60						.000005	8.60
8.70						.000004	8.70
8.80						.000003	8.80
8.90						.000003	8.90
9.00						.000002	9.00
9.10							9.10
9.20							9.20
9.30							9.30
9.40							9.40
9.50							9.50
9.60							9.60
9.70							9.70
9.80							9.80
9.90							9.90

	SKEWNESS						
t	.6	.7	.8	.9	1.0	1.1	t
5.00	.010631	.009320	.008162	.007164	.006315	.005598	5.00
5.10	.009292	.008236	.007273	.006424	.005690	.005061	5.10
5.20	.008075	.007240	.006451	.005737	.005107	.004550	5.20
5.30	.006980	.006334	.005697	.005103	.004568	.004096	5.30
5.40	.006004	.005517	.005010	.004523	.004074	.003670	5.40
5.50	.005140	.004785	.004391	.003996	.003622	.003279	5.50
5.60	.004382	.004135	.003835	.003520	.003212	.002923	5.60
5.70	.003721	.003560	.003339	.003091	.002841	.002600	5.70
5.80	.003148	.003055	.002898	.002708	.002507	.002308	5.80
5.90	.002654	.002614	.002509	.002366	.002207	.002045	5.90
6.00	.002230	.002230	.002166	.002063	.001940	.001809	6.00
6.10	.001867	.001897	.001865	.001795	.001701	.001597	6.10
6.20	.001559	.001609	.001603	.001558	.001490	.001408	6.20
6.30	.001298	.001362	.001374	.001350	.001302	.001239	6.30
6.40	.001078	.001150	.001176	.001168	.001136	.001090	6.40
6.50	.000893	.000968	.001004	.001008	.000920	.000957	6.50
6.60	.000737	.000814	.000856	.000869	.000861	.000839	6.60
6.70	.000608	.000683	.000728	.000748	.000749	.000735	6.70
6.80	.000500	.000572	.000618	.000643	.000650	.000643	6.80
6.90	.000410	.000478	.000524	.000552	.000563	.000562	6.90
7.00	.000336	.000398	.000444	.000473	.000487	.000490	7.00
7.10	.000274	.000332	.000375	.000405	.000421	.000428	7.10
7.20	.000224	.000276	.000317	.000346	.000364	.000373	7.20
7.30	.000182	.000229	.000267	.000295	.000314	.000324	7.30
7.40	.000148	.000190	.000225	.000252	.000271	.000282	7.40
7.50	.000120	.000157	.000189	.000215	.000233	.000245	7.50
7.60	.000097	.000130	.000159	.000183	.000201	.000213	7.60
7.70	.000078	.000107	.000133	.000155	.000172	.000185	7.70
7.80	.000063	.000088	.000112	.000132	.000148	.000160	7.80
7.90	.000051	.000073	.000093	.000112	.000127	.000139	7.90
8.00	.000041	.000060	.000078	.000095	.000109	.000120	8.00
8.10	.000033	.000049	.000065	.000080	.000093	.000104	8.10
8.20	.000026	.000040	.000054	.000068	.000080	.000090	8.20
8.30	.000021	.000033	.000045	.000057	.000068	.000078	8.30
8.40	.000017	.000027	.000038	.000049	.000059	.000067	8.40
8.50	.000014	.000022	.000031	.000041	.000050	.000058	8.50
8.60	.000011	.000018	.000026	.000035	.000043	.000050	8.60
8.70	.000009	.000015	.000022	.000029	.000036	.000043	8.70
8.80	.000007	.000012	.000018	.000025	.000031	.000037	8.80
8.90	.000005	.000010	.000015	.000021	.000027	.000032	8.90
9.00	.000004	.000008	.000012	.000017	.000023	.000028	9.00
9.10	.000003	.000006	.000010	.000015	.000019	.000024	9.10
9.20	.000003	.000005	.000008	.000012	.000016	.000020	9.20
9.30	.000002	.000004	.000007	.000010	.000014	.000018	9.30
9.40	.000002	.000003	.000006	.000009	.000012	.000015	9.40
9.50	.000001	.000003	.000005	.000007	.000010	.000013	9.50
9.60	.000001	.000002	.000004	.000006	.000009	.000011	9.60
9.70		.000002	.000003	.000005	.000007	.000010	9.70
9.80		.000001	.000003	.000004	.000006	.000008	9.80
9.90		.000001	.000002	.000004	.000005	.000007	9.90

t	SKEWNESS						t
	.0	.1	.2	.3	.4	.5	
10.00							10.00
10.10							10.10
10.20							10.20
10.30							10.30
10.40							10.40
10.50							10.50
10.60							10.60
10.70							10.70
10.80							10.80
10.90							10.90
11.00							11.00
11.10							11.10
11.20							11.20
11.30							11.30
11.40							11.40
11.50							11.50
11.60							11.60
11.70							11.70
11.80							11.80
11.90							11.90
12.00							12.00
12.10							12.10
12.20							12.20
12.30							12.30
12.40							12.40
12.50							12.50
12.60							12.60
12.70							12.70
12.80							12.80
12.90							12.90
13.00							13.00
13.10							13.10
13.20							13.20
13.30							13.30
13.40							13.40
13.50							13.50
13.60							13.60
13.70							13.70
13.80							13.80
13.90							13.90
14.00							14.00
14.10							14.10
14.20							14.20
14.30							14.30
14.40							14.40
14.50							14.50
14.60							14.60
14.70							14.70
14.80							14.80
14.90							14.90

	SKEWNESS						
t	.6	.7	.8	.9	1.0	1.1	t
10.00		.000001	.000002	.000003	.000004	.000006	10.00
10.10		.000001	.000001	.000002	.000004	.000005	10.10
10.20		.000001	.000001	.000002	.000003	.000004	10.20
10.30			.000001	.000002	.000003	.000004	10.30
10.40			.000001	.000001	.000002	.000003	10.40
10.50			.000001	.000001	.000002	.000003	10.50
10.60			.000001	.000001	.000002	.000002	10.60
10.70				.000001	.000001	.000002	10.70
10.80				.000001	.000001	.000002	10.80
10.90				.000001	.000001	.000001	10.90
11.00					.000001	.000001	11.00
11.10					.000001	.000001	11.10
11.20					.000001	.000001	11.20
11.30					.000001	.000001	11.30
11.40						.000001	11.40
11.50						.000001	11.50
11.60							11.60
11.70							11.70
11.80							11.80
11.90							11.90
12.00							12.00
12.10							12.10
12.20							12.20
12.30							12.30
12.40							12.40
12.50							12.50
12.60							12.60
12.70							12.70
12.80							12.80
12.90							12.90
13.00							13.00
13.10							13.10
13.20							13.20
13.30							13.30
13.40							13.40
13.50							13.50
13.60							13.60
13.70							13.70
13.80							13.80
13.90							13.90
14.00							14.00
14.10							14.10
14.20							14.20
14.30							14.30
14.40							14.40
14.50							14.50
14.60							14.60
14.70							14.70
14.80							14.80
14.90							14.90